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Abstract

Context: Trait composition has inspired new research in the area of code reuse for object oriented (OO) languages. One of the main advantages of this kind of composition is that it makes possible to separate subtyping from subclassing; which is good for code-reuse, design and reasoning [15]. However, handling of state within traits is difficult, verbose or inelegant.

Inquiry: We identify the *this-leaking problem* as the fundamental limitation that prevents the separation of subtyping from subclassing in conventional OO languages. We explain that the concept of trait composition addresses this problem, by distinguishing code designed for use (as a type) from code designed for reuse (i.e. inherited). We are aware of at least 3 concrete independently designed research languages following this methodology: TraitRecordJ [6], Package Templates [26] and DeepFJig [16].

Approach: In this paper, we design 42_{μ} , a new language, where we improve use and reuse and support the **This** type and family polymorphism by distinguishing code designed for use from code designed for reuse. In this way 42_{μ} synthesise the 3 approaches above, and improves them with *abstract state operations*: a new elegant way to handle state composition in trait based languages.

Knowledge and Grounding: Using case studies, we show that 42_{μ} 's model of traits with abstract state operations is more usable and compact than prior work. We formalise our work and prove that type errors cannot arise from composing well typed code.

Importance: This work is the logical core of the programming language 42. This shows that the ideas presented in this paper can be applicable to a full general purpose language. This form of composition is very flexible and could be used in many new languages.

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1 Introduction

In Java, C++, Scala and C#, subclassing implies subtyping. A Java subclass declaration, such as **class** A **extends** B {} does two things at the same time: it **inherits** code from B; and it creates a subtype of B. Therefore a subclass must *always* be a subtype of the extended class. Such design choice where subclassing implies subtyping is not universally accepted. Historically, there has been a lot of focus on separating subtyping from subclassing [15]. This separation is claimed to be good for code-reuse, design and reasoning. There are at least two distinct situations where the separation of subtyping and subclassing is helpful.

- Allowing inheritance/reuse even when subtyping is impossible: Situations where inheritance is desirable are prevented by the enforced subtyping relation. A well-known example are the so-called *binary methods* [II, I5]. For example, consider a class Point with a method Point sum(Point o){return new Point(x+o.x,y+o.y);}. Can we reuse the Point code so that ColorPoint.sum would take and return a ColorPoint? In Java/C# declaring class ColorPoint extends Point{..} would result in sum still taking a Point and returning a Point. Moreover, manually redeclaring a ColorPoint sum(ColorPoint that) would just induce overloading, not overriding. In this case we would like to have inheritance, but we cannot have (sound) subtyping.
- **Preventing unintended subtyping:** For certain classes we would like to inherit code without creating a subtype even if, from the typing point of view, subtyping is still sound. A typical example [29] is *Sets* and *Bags*. Bag implementations can often inherit from Set implementations, and the interfaces of the two collection types are similar and type compatible. However, from the logical point-of-view a Bag is *not a subtype* of a Set.

Structural typing [15] may deal with the first situation, but not the second. Since structural subtyping accounts for the types of the methods only, a Bag would be a subtype of a Set if the two interfaces are type compatible. For dealing with the second situation, nominal subtyping is preferable: an explicit subtyping relation must be signalled by the programmer. Thus if subtyping is not desired, the programmer can simply **not** declare a subtyping relationship.

While there is no problem in subtyping without subclassing, in most nominal OO languages subclassing fundamentally implies subtyping. This is because of what we call the *this-leaking problem*, illustrated by the following (Java) code, where method A.ma passes **this** as A to Utils.m. This code is correct, and there is no subtyping/subclassing.

```
1 class A{ int ma(){ return Utils.m(this); } }
```

```
2 class Utils{ static int m(A a){..} }
```

Now, lets add a class B

```
class B extends A{ int mb(){return this.ma();} }
```

We can see an invocation of A.ma inside B.mb, where the self-reference **this** is of type B. The execution will eventually call Utils.m with an instance of B. However, *this can be correct only if* B *is a subtype of* A.

Suppose Java code-reuse (the **extends** keyword) did not introduce subtyping¹: then an invocation of B.mb would result in a run-time type error. The problem is that the self-reference **this** in class B has type B. Thus, when **this** is passed as an argument to the method Utils.m (as a result of the invocation of B.mb), it will have a type that is incompatible with the expected argument of type A. Therefore, every OO language with the minimal features exposed in the example (using **this**, **extends** and method calls) is forced to accept that subclassing implies subtyping.

What the *this-leaking problem* shows is that adopting a more flexible nominally typed OO model where subclassing does not imply subtyping is not trivial: a more substantial change in the language design is necessary. In essence we believe that in languages like Java, classes do too many things at once. In particular they act both as units of *use* and *reuse*: classes can be *use*d as types and can be instantiated; classes can also be subclassed to provide *reuse* of code. We are aware of at least 3 independently designed research languages that address the *this-leaking problem*:

- In **TraitRecordJ (TR)** [6, 7, 8] each construct has a single responsibility: classes instantiate objects, interfaces induce types, records express state, and traits are reuse units.
- **Package Templates (PT)** [2, 3, 26]: an extension of (full) Java where new packages can be "synthesized" by mixing and integrating code templates. Such "synthesized" packages can be used for code reuse without inducing subtyping.
- **DeepFJig(DJ)** [16, 28, 37] is a module composition language where nested classes with the same name are recursively composed.

This paper shows a simple language design, called 42_{μ} , addressing the *this-leaking problem* and decoupling subtyping from inheritance. We build on traits to distinguish code designed for *use* from code designed for *reuse*. We synthesize and simplify the best ideas from those 3 very different designs, and couple them with an elegant novel approach to state and self instantiation in traits that avoids the complexities and redundancies introduced by fields and their initialisation.

In 42_{μ} , there are two separate concepts: classes and traits [18]. Classes are meant for code *use*, and cannot be inherited/extended. Classes in 42_{μ} are like final classes in Java, and can be used as types and as object factories. Traits are meant for code *reuse* only: multiple traits can be composed to form a class. However, traits cannot be instantiated or used as types. This allows fine-grained control of subtyping while handling examples like Set/Bag.

In 42_{μ} , as in many module composition languages [1], all methods can be abstract, including static ones. Moreover, module composition can be used to make an already implemented method abstract. Thus, as for dynamic dispatch, the behaviour of a method call is never set in stone. We will show how in 42_{μ} , state is induced by an implicit fixpoint operation over abstract methods, where an abstract static method can

^I C++ allows "extending privately"; this is not what we mean by not introducing subtyping: in C++ it is a limitation over subtyping visibility, not over subtyping itself. Indeed, the former example would be *accepted* even if B were to "privately extends" A

perform the role of a constructor. This allows handling examples like Point/ColorPoint in a natural way, without requiring code duplication.

Our design brings several benefits. In particular, Family Polymorphism [20] is radically simpler to support soundly. This is already clear in the 3 lines of research above, and is even more outstanding in the clean 42_{μ} model.

We first focus on an example-driven presentation to illustrate how to improve use and reuse. In Appendix A, we then provide a compact formalization. The hard technical aspects of the semantics have been studied in previous work [2, 3, 6, 7, 8, 16, 26, 28, 37]; the design of 42_{μ} synthesizes some of those concepts. The design ideas have been implemented in the full 42 language, which supports all the examples we show in the paper, and is available at: http://l42.is. Work on 42 is now slowly reaching maturity after about 5 years of intense research and development. The current implementation is now robust enough to create realistic medium sized programs running on the JVM, and the standard library consists of over 10000 lines of 42 code.

In summary, our contributions are:

- We identify the this-leaking problem, that makes separating inheritance and subtyping difficult.
- We synthesize the key ideas of previous designs that solve the this-leaking problem into a novel and minimalistic language design. This language is the core logic of the language 42, and all the examples in this paper can be encoded as valid 42 programs. This design improves both code use and code reuse.
- We propose a clean and elegant approach to the handling of state in a trait based language.
- We illustrate how 42_μ, extended with nested classes, enables a powerful (but at the same time simple) form of family polymorphism.
- We show the simplicity of our approach by providing a compact 1 page formalization (in Appendix A).
- We perform 3 case studies, comparing our work with other approaches, and we collect clear data showing that we can express the same examples in a cleaner and more modular manner.

2 The Design of 42_{μ} : Separating Use and Reuse

2.1 Classes in 42_{μ} : a mechanism for code use

Consider the example of Section I rewritten in 42_{μ} , introducing classes Utils and A:

```
1 A={ method int ma(){ return Utils.m(this); } }
```

```
2 Utils={ static method int m(A a){ return ..; } }
```

Classes in 42_{μ} use a different declaration style compared to Java: there is no **class** keyword, and an equals sign separates the class name (which must always start with an uppercase letter) and the class implementation, which is used to specify the definitions of the class. In our example, in the class declaration for A, the name of the class is A and the code literal associated with the class is '{ **method int** ma(){**return** Utils.m(**this**);}}'

and it contains the method ma(). In the 42_{μ} code above, there is no way to add a class B reusing the code of A: class A (uppercase) is designed for code *use* and not *reuse*. Indeed, a noticeable difference with Java is the absence of the **extends** keyword. 42_{μ} classes are roughly equivalent to final classes in Java. This means that there is actually no subclassing. Thus, unlike the Java code, introducing a subclass B is not possible. This may seem like a severe restriction, but 42_{μ} has a different mechanism for *code-reuse* that is more appropriate when *code-reuse* is intended.

2.2 Traits in 42_{μ} : a mechanism for code reuse

Traits in 42_{μ} cannot be instantiated and do not introduce new types. However they provide code reuse. Trait declarations look very much like class declarations, but trait names start with a lowercase letter (even syntactically they can not be used as types).

```
1 Utils={ static method int m(A a){return ...} }
2 ta={ method int ma(){return Utils.m(this);}} //type error
3 A=Use ta
```

Here ta is a trait intended to replace the original class A so that the code of the method ma can be reused. Then the class A is created by reusing the code from the trait ta, introduced by the keyword **Use**. Note that **Use** expressions cannot contain class names: only trait names are allowed. *Referring to a trait is the only way to induce code reuse*.

The crucial point is the call Utils.m(this) inside trait ta: the corresponding call in the Java code is correct since Java guarantees that such occurrence of this will be a subtype of A everywhere it is reused. In 42_{μ} the type of this in ta has no relationship to the type A; thus the code Utils.m(this) is illtyped.

The following second attempt would not work either:

```
1 Utils={ static method int m(ta a){return ...}} //syntax error
```

```
2 ta={ method int ma(){return Utils.m(this);}}
```

```
3 A=Use ta
```

ta is not a type in the first place, since it is a (lowercase) trait name. Indeed, trait names can only be used in **Use** expressions, and thus they can not appear in method bodies or type annotations. In this way, the code of a trait can stay agnostic of its name. This is one of the key design decisions in 42_{μ} : traits can be reused in multiple places, and their code can be seen under multiple types. In 42_{μ} , *interfaces are the only way to obtain subtyping*. As shown in the code below, interfaces are special kinds of code literals, where all the methods are abstract. Thus, to model the original Java example, we need an interface capturing the commonalities between A and B:

```
1 IA={interface method int ma()} //interface with abstract method
2 Utils={static method int m(IA a){return ...} }
3 ta={implements IA //This line is the core of the solution
4 method int ma(){return Utils.m(this);}}
5 A=Use ta
```

This code works: Utils relies on interface IA and the trait ta implements IA. Any class reusing ta will contain the code of ta, including the **implements** IA subtyping declaration; thus any class reusing ta will be a subtype of IA. Therefore, while typechecking Utils.m(this) we can assume this<: IA. It is also possible to add a class B as follows:

```
B=Use ta, { method int mb(){return this.ma();} }
```

This also works. B reuses the code of ta, but has no knowledge of A. Since B reuses ta, and ta implements IA, B also implements IA.

Later, in Appendix A we will provide the type system. Here notice that the former declaration of B is correct even if no method called ma is explicitly declared. DJ and TR would instead require explicitly declaring an abstract ma method:

```
1 B=Use ta, { method int ma() //not required by us
2 method int mb(){return this.ma();} }
```

In 42_{μ} , methods are directly accessible from ta, exactly as in the Java equivalent

```
class B extends A{ int mb(){return this.ma();} }
```

where method m_a is imported from A. This concept is natural for a Java programmer, but was not supported in previous work [8, 16]. Those works require all dependencies in code literals to be explicitly declared, so that the code literal is self-contained; in this way it can be typed in isolation before flattening. However, this results in many redundant abstract method declarations.

Semantics of Use: The semantics of traits is defined with flattening, which is simple to formalize and understand. However, if implemented naively, flattening may cause a lot of bytecode duplication. The 'delegation semantics' [27], is a proposed alternative semantic model for traits that is observationally equivalent to flattening but does not require bytecode duplication. The formalism presented here will rely on simple flattening, but we expect the techniques of [27] would be useful to produce an efficient implementation in term of bytecode space. With the flattening semantics A and B are equivalent to the inlined code of all used traits.

```
1 A=Use ta
2 B=Use ta, { method int mb(){return this.ma();} }
3 //equivalent to
4 A={implements IA method int ma(){return Utils.m(this);}}
5 B={implements IA
6 method int ma(){return Utils.m(this);}
7 method int mb(){return this.ma();} }
```

In the resulting code, there is no mention of the trait ta. Information about codereuse/inheritance is a private implementation detail of A and B; while subtyping is part of the class interface. This position has been defended by Bracha [10]: the choice of inheriting behaviour should be in the hands of the programmer; if a method implementation is not appropriate, such method can be overridden. If too many methods do not provide an appropriate behaviour, inheriting code from another location or implementing the behaviour from scratch may also be considered. This should not impact the interface exposed to the user, otherwise the programmer may be unable to change their implementation decisions at a later time. In summary, to leak **this** in 42_{μ} , either code reuse is disallowed, or an appropriate interface (IA in this case) must be implemented. We believe the code with IA better transmits programmer intention. Some readers may instead see requiring IA as a cost of our approach. Even from this point of view, such cost is counter balanced by the very natural and simple support for code reuse, 'This' type and (in the extensions with nested classes seen later) family polymorphism. The syntactic cost of introducing new names can be reduced with some syntactic sugar, as explained in Appendix B.1.

3 Improving Use

To illustrate how 42_{μ} improves the use of classes, we model a simplified version of Set and Bag collections first in Java, and then in 42_{μ} . The benefit of 42_{μ} is that we get reuse without introducing subtyping between Bags and Sets. As shown below, this improves the use of Bags by eliminating logical errors arising from incorrect subtyping relations that are allowed in the Java solution.

3.1 Sets and Bags in Java: the need for code reuse without subtyping

An iconic example on why connecting inheritance/code reuse and subtpying is problematic is provided by LaLonde [29]. A reasonable implementation for a set is easy to extend into a Bag by keeping track of how many times an element occurs. We just add some state and override a few methods. For example in Java one could have:

```
1 class Set {..//usual hashmap implementation
2 private Elem[] hashMap;
3 void put(Elem e){..}
4 boolean isIn(Elem e){..}}
5 class Bag extends Set{ ..//for each element in the hash map,
6 private int[] countMap;// keep track of how many occurrences are in the bag
aOverride void put(Elem e){..}
8 int howManyTimes(Elem e){..}}
```

Coding Bag in this way avoids a lot of code duplication, but we induced unintended subtyping! Since subclassing implies subtyping, our code breaks the Liskov substitution principle (LSP) [30]: not all bags are sets!² Indeed, the following is allowed:

1 Set mySet=new Bag(); //OK for the type system but not for LSP

This encumbers the programmer: to avoid conceptual errors that are not captured by the type system, they have to *use* Bag very carefully.

A (broken) attempt to fix the Problem in Java: One could *retroactively* fix this problem by introducing AbstractSetOrBag and making both Bag and Set inherit from it:

```
1 abstract class AbstractSetOrBag {/*old set code goes here*/}
2 class Set extends AbstractSetOrBag {} //empty body
3 class Bag extends AbstractSetOrBag {/*old bag code goes here*/}
4 ...
5 //AbstractSetOrBag type not designed to be used.
6 AbstractSetOrBag unexpected=new Bag();
```

This looks unnatural, since Set would extend AbstractSetOrBag without adding anything, and we would be surprised to find a use of the type AbstractSetOrBag. Worst, if we are to constantly apply this mentally, we would introduce a very high number of abstract classes that are not supposed to be used as types. Those classes would clutter the public interface of our classes and the project as a whole. A *use*able API should provide only the information relevant to the client. In our example, the information Set<:AbstractSetOrBag would be present in the public interface of the class Set, but

² The LSP is often broken in real programs because of the need of inheritance: the LSP allows only refinement not extension. Traits provide extension without breaking the LSP.

such information is not needed to use the class properly! Moreover, the original problem is not really solved, but only moved further away. For example, one day we may need bags that can only store up to 5 copies of the same element. We are now at the starting point again:

- either we insert class Bag5 extends Bag and we break the LSP;
- or we duplicate the code of the Bag implementation with minimal adjustments in class Bag5 extends AbstractSetOrBag;
- or we introduce an abstract class BagN extends AbstractSetOrBag and class Bag5 extends BagN and we modify Bag so that class Bag extends BagN. Note that this last solution is changing the public interface of the formerly released Bag class, and this may even break backwards-compatibility (if a client program was using reflection, for example).

3.2 Sets and Bags in 42_{μ}

Instead, in 42_{μ} , if we were to originally declare

```
1 Set={/*set implementation*/}
```

Then our code would be impossible to reuse in the first place for any user of our library. We consider this an advantage, since unintended code reuse runs into underdocumented behaviour nearly all the time!³ If the designer of the Set class wishes to make it reusable, they can do it explicitly by providing a set trait:

```
1 set={/*set implementation*/}
```

2 Set=**Use** set

Since set can never be used as a type, there is no reason to give it a fancy-future-aware name like AbstractSetOrBag. When bag is added, the code will look like

```
1 set={/*set implementation*/}
2 Set=Use set
3 Bag= Use set, {/*bag implementation*/}
```

or

```
1 set={/*set implementation*/}
2 Set=Use set
3 bag=Use set, {/*bag implementation*/}
```

4 Bag=**Use** bag

Notice how, thanks to flattening, the resulting code for Bag is identical in both versions and, as shown in Section 2, there is no trace of trait bag at run time. Thus if we are the developers of bags, we can temporarily go for the first version. Then, when for example we need to add Bag5 as discussed before, we can introduce the bag trait without adding new undesired complexity for our old clients.

³ See "Design and document for inheritance or else prohibit it" [9]: the self use of public methods is rarely documented, thus is hard to understand the effects of overriding a library method.

4 Improving Reuse

To illustrate how 42_{μ} improves reuse, we show a novel approach to smoothly integrating state and traits: a challenging problem that has limited the flexibility of traits and reuse in the past. The idea of flattening is elegant and successful in module composition languages [1] and several trait models [5, 8, 18, 28]. Flattening is elegant in these two settings since traits (or modules) only have one kind of member: methods (or functions). In this way flattening is defined as simply collecting all members from all used traits (or composed modules), where methods with same name and type signature are summed into a single one. At most one of those summed methods can have a body, which will be propagated into the result. However the research community is struggling to make it work with object state (constructors and fields) while achieving the following goals:

- managing fields in a way that borrows the elegance of summing methods;
- actually initializing objects, leaving no null fields;
- making it easy to add new fields;
- allowing self instantiation: a trait method can instantiate the class using it.

An in-depth discussion on how such goals are difficult to achieve and how they have been challenged in the existing literature is available in Section 7.3.

4.1 State of the art

We first present the state of the art solution: traits have only methods but classes also have fields and constructors. The idea is that the trait code just uses getter/setters/factories, while leaving classes to finally define the fields/constructors. That is, in this state of the art solution, classes have a richer syntax than traits, allowing declaration for fields and constructors.

Points: Consider two traits dealing with *point* objects with coordinates x and y.

```
1 //idealized state of the art trait language, not 42
2 pointSum= { method int x() method int y()//getters
3
   static method This of(int x,int y)//factory method
4
   method This sum(This that){//sum code
     return This.of(this.x()+that.x(),this.y()+that.y());//self instantiation
5
6
   }}
7 pointMul= { method int x() method int y()//repeating getters
   static method This of(int x,int y)//repeating factory
8
   method This mul(This that){//multiplication code
9
     return This.of(this.x()*that.x(),this.y()*that.y());
10
11
    }}
```

The first trait provides a *binary method* that adds the point object to another point to return a new point. The second trait provides multiplication. In this code all the operations dealing with state are represented as *abstract methods*. Notice the abstract **static method This** of(..) which acts as a factory/constructor for points. As for instance methods, static methods are late bound: flattening can provide an implementation for them. Thus, in 42_{μ} they can be abstract, and abstract static methods are similar to the concept of member functions in the module composition setting [I].

Following the traditional model of traits and classes common in literature [18], we can compose the two traits, by *adding glue-code* to implement methods x, y and of. This approach is verbose but very powerful, as illustrated by Wang et al. [40].

```
1 //idealized state of the art trait language, not 42
2 class PointAlgebra=Use pointSum,pointMul, {//not 42 code
3    int x int y//unsatisfactory state of the art solution
4    constructor PointAlgebra(int x, int y){ this.x=x this.y=y }
5    method int x(){return x;}//repetitive code
6    method int y(){return y;}// in traits terminology, this is all "glue code"
7    static method This of(int x, int y){return new PointAlgebra(x,y);}
8  }
```

With a slightly different syntax, this approach is available in both Scala and Rust, and they both require glue code. It has some advantages, but also disadvantages:

- Advantages: This approach is associative and commutative, even self instantiation can be allowed if the trait requires a static method returning This. The class will then implement the methods returning This by forwarding a call to the constructor.
- Disadvantages: The class needs to handle all the state, even state conceptually
 private to a trait. Moreover, writing such obvious code to close the state/fixpoint in
 the class with the constructors and fields and getter/setters and factories is tedious
 and error prone; such code could be automatically generated [40].

4.2 Our proposed approach to State: Coherent Classes

In 42_{μ} there is no need to write down constructors and fields. In fact, in 42_{μ} there is not even syntax for those constructs! The intuition is that a class where *all* abstract methods can be seen as field getters, setters, or factories, is a *coherent* class. In most other languages, a class is abstract if it has abstract methods. Instead, we call a class abstract only when the set of abstract methods are not coherent. That is, the abstract methods cannot be automatically recognised as factory, getters or setters. Methods recognised as factory, getters and setters are called *abstract state operations*.

A definition of coherent classes is given next, and is formally modelled in Appendix A:

- A class with no abstract methods is coherent (just like Java Math, for example). Such classes have no instances and are only useful for calling static methods.
- A class with a single abstract **static** method returning **This** and with parameters $T_1 x_1, \ldots, T_n x_n$ is coherent if all the other abstract methods can be seen as *abstract state operations* over one of x_1, \ldots, x_n . That is:
 - A method $T_i x_i$ () is interpreted as an abstract state method: a getter for x_i .
 - A method **void** $x_i(T_i \text{ that})$ is a setter for x_i .

Note how the single, abstract static method acts as a *factory method*. The signature of the factory method plays an important role, since abstract state operations are identified by using the names of the factory method arguments. The idea of creating objects in a single atomic step by providing a value for all their fields is well explored (such as with primary constructors in Scala) and does not limit the freedom of programmers to specify personalised initialisation strategies. A static method can freely compute concrete field values before creating objects. Appendix B.4 discusses usability implications of this pattern.

While getters and setters are fundamental operations, it is possible to support more operations. For example:

- method This withX(int that) may create a new instance that is like this except that field x now has value that. Those kinds of methods performs functional field updates and are called withers.
- method This clone() may do a shallow clone of the object.

The concept of 'abstract state operations' is novel, and we think it is a promising area for further research. The work by Wang et al. [40] explores a particular set of such abstract state operations, but we suspect there are more unexplored possible options that could be even more beneficial.

Points in 42_{μ} : In 42_{μ} and with our approach to handle the state, pointSum and pointMul can indeed be directly composed. This works because the resulting class is coherent.

```
1 PointAlgebra= Use pointSum,pointMul //no glue code needed
```

Improved solution: So far the current solution still repeats the abstract methods x, y and of. Moreover, in addition to sum and mul we may want many operations over points. It is possible to improve reuse and not repeat such declaration by abstracting the common declaration into a trait p:

```
1 p= { method int x() method int y()
 2
   static method This of(int x,int y)
   }
 3
 4 pointSum= Use p, {
    method This sum(This that){
 5
 6
     return This.of(this.x()+that.x(),this.y()+that.y());
   }}
 7
8 pointMul= Use p, {
   method This mul(This that){
9
     return This.of(this.x()*that.x(),this.y()*that.y());
11 }}
12 pointDiv= ...
13 PointAlgebra= Use pointSum, pointMul, pointDiv,...
```

Now the code is fully modularized, that is: each trait defines exactly one method and contains its abstract dependencies. In this way it can be modularly composed with any code requiring such a method.

Case Study 1: In order to evaluate our approach we performed a case study: we consider 4 different operations Sum, Subtraction, Multiplication and Division. These operations can be combined in 16 different ways. We wrote this example in four different styles: (a) Java7 (115 *lines*), (b) Classless Java (82 *lines*), (c) Scala (81 *lines*) and (d) 42_{μ} (32 *lines*).⁴ We chose Classless Java [40] since it is a novel approach allowing Java8 default interface methods to encode traits in Java. We then chose Java7, that lacks the features needed to encode traits, to show the impact of this feature. Finally, the comparison with Scala is interesting since it has good support for

⁴ Since we want to focus on the actual code, while counting line numbers we *omit* empty lines and lines containing only open/closed parenthesis/braces.

traits, and using abstract types, it is possible to support the '**This**' type. Rust is similar to Scala in this regard; we believe we would get similar results by comparing against either Scala or Rust.

Language	Lines of code	N. of members	N. classes/traits
Java7	115 = 6 + 5 * 4 + 7 * 6 + 9 * 4 + 11	50	16
Classless Java	82 = 3 + 3 * 4 + 5 * 6 + 7 * 4 + 9	34	16
Scala	81 = 5 + 3 * 4 + 4 * 16	40	21 = 16 + 4 + 1
42_{μ}	32 = 4 + 3 * 4 + 1 * 16	7	21 = 16 + 4 + 1

We observed that in Java7 we had to duplicate⁵ 28 method bodies across the 16 classes. Of these, 11 method bodies were duplicated because Java does not support multiple inheritance and the remaining 17 bodies had to be duplicated to ensure that the right type is returned by the method. Those could be avoided if Java supported the '**This**' type. On the other hand, the solution in 42_{μ} was much more compact since we could efficiently reuse traits (this is why the number of top-level concepts in 42_{μ} was larger i.e. 21 due to the presence of traits in this solution). In detail, Java required 6 lines for the initial Point class, 5 lines for each of the 4 arithmetic operations, 7 lines for each of the 6 combinations of two different operations, 9 lines for each of the 4 combinations of three different operations and finally 11 lines for the class with all four operations. The solution in Classless Java was slightly smaller than Java7, but was still longer than the 42_{μ} solution: it still had to redefine the sum, sub and other operations in each of the classes. Here the limited support for the '**This**' type is to blame, thus Classless Java also has 28 duplicated method bodies.

Finally, we compare it with a Scala solution. There is no need for duplicate method bodies in Scala. However, for '**This**' instantiation we need to define abstract methods, that will be implemented in the concrete classes. The Scala solution has the same exact advantages of our proposed solution, and the declaration of the trait is about the same size: 5 (point state) +3 * 4 (point operations). However the glue code (the code needed to compose the traits into usable classes) is quite costly: 4 lines for each of the 16 cases. In 42_{μ} a single line for each case is sufficient.

This example is the best-case scenario for 42_{μ} : where a maximum level of reuse is required since we considered the case where all the 16 permutations needed to be materialized in the code. In all our case studies, to make a meaningful comparison, we formatted all code in a readable and consistent manner; on the other hand for space limitations, the code snippets presented in the article are formatted for compactness.

4.3 State Extensibility

Programmers may want to extend points with more state. For example they may want to add colors to the points. A first attempt at doing this would be:

```
1 colored= { method Color color() }
```

² CPoint= Use pointSum, colored //Fails: class not coherent

⁵ A duplicate body is repetition of identical code (may have different types in its scope/environment). The first occurrence is not counted.

This first attempt does not work: the abstract color method is not a getter for any of the parameters of **static method This** of(**int** x,**int** y). A solution is to provide a richer factory:

```
1 CPoint= Use pointSum,colored,{
```

```
static method This of(int x,int y){return This.of(x,y,Color.of(/*red*/));}
```

```
static method This of(int x,int y,Color color) }
```

where we assume support for overloading based on different numbers of parameters. This is a reasonable solution, however the method CPoint.sum resets the color to red: we call the of(int, int) method, that now delegates to of(int, int, Color) by passing red as the default field value. What should be the behaviour in this case? If our abstract state supports withers, we can use this.withX(newX).withY(newY), instead of writing This.of(...), in order to preserve the color from this. This solution is better but still not satisfactory since the color from that is ignored.

A better design: We can design trait p for reuse and extensibility by adding an abstract merge(**This**) method as an extensibility hook; colored can now define color merging. Using withers we can merge colors, or any other kind of state following this pattern.

```
1 p= { method int x() method int y() //getters
    method This withX(int that) method This withY(int that)//withers
 2
 3
   static method This of(int x, int y)
    method This merge(This that) //new method merge!
 4
 5
   }
 6 pointSum= Use p, {
    method This sum(This that){
 7
8
     return this.merge(that).withX(this.x()+that.x()).withY(this.y()+that.y());
9
   }}
10 colored= {method Color color()
11 method This withColor(Color that)
12 method This merge(This that){ //how to merge color handled here
    return this.withColor(this.color().mix(that.color());
13
14 }}
15 CPoint= /*as before*/
```

Independent Extensibility: Of course, quite frequently there can be multiple independent extensions [41] that need to be composed. Lets suppose that we could have a notion of flavored as well. In order to compose colored with flavored we would need to compose their respective merge operations. To this aim **Use** is not sufficient. To combine the implementation of two different implementation of methods, we introduce an operator called **super**, that makes a method abstract and moves the implementation to another name. This is very useful to implement super calls and to compose conflicting implementations. Consider the simple flavored trait:

```
1 flavored= {
2 method Flavor flavor() //very similar to colored
3 method This withFlavor(Flavor that)
4 method This merge(This that){ //merging flavors handled here
5 return this.withFlavor(that.flavor());}//inherits "that" flavor
```

In order to merge colored and flavored we use **super** to introduce method selectors _1merge and _2merge to refer to the version of merge as defined in the first/second element of **use**.

```
1 FCPoint= Use
  colored[super merge as _1merge], //this leaves merge as an abstract method, and
2
3 flavored[super merge as _2merge],//copies the bodies into _1merge and _2merge
4 pointSum,{
    static method This of(int x,int y){
5
      return This.of(x,y,Color.of(/*red*/),Flavor.none());}
6
7
    static method This of(int x, int y,Color color,Flavor flavor)
8
    method This merge(This that){//merge conflict is solved
      return this._1merge(that)._2merge(that);}//by calling the two versions
9
     }
```

Note how we are relying on the fact that the code literal does not need to be complete, thus we can just call _1merge and _2merge without declaring their abstract signature explicitly.

In this last example, when we tried to obtain state extensibility, we refactored the code to introduce the merge(**This**) method. This suggests that we had to anticipate the need for state extensibility in order to design our original code. As illustrated by the following example, we can instead rely on the **super** operator to inject the merge(**This**) method when needed.

```
1 p=/*as originally designed: no merge*/
2 pointSum=/*as originally designed: no merge*/
3 merge={method This merge(This that)}
4 pointSumMerge=Use merge, pointSum[super sum as _1sum], {
5 method This sum(This that){return this.merge(that)._1sum(that);}}
6 colored=/*as before, with merge implementation*/
7 CPoint= /*as before, but using pointSumMerge*/
```

Case Study 2: To understand how easy it is to extend the state in this way we compare the former code with an equivalent version in Java. For this example, in Java we encode Point with the fields but no operations, PointSum reuses Point adding a functional sum operation, CPoint reuses PointSum with a Color field and FCPoint reuses CPoint with a Flavour field. This second case study represents a *worst case scenario* for 42_{μ} against Java because we model just a single chain of reuse, easily supported in plain Java by single inheritance. Like the previous experiment, we still found that the Java solution was longer (47 lines) than that in 42_{μ} (33 lines). This is caused by the absence of support for the 'This' type, where the withers in each of the CPoint/FCpoint classes had to be repeated to make sure that the returned type will be correct (the number of members in Java were 27 while 24 (3 less) in 42_{μ}).

Complex patterns in Java⁶ allow supporting the '**This**' type and '**This**' type instantiation but they require a lot of set-up code. We experimented with those patterns, but it soon became very clear that the resulting code of this approach would have been even larger; albeit without duplicated code. Note how the Java code is less modular than the 42_{μ} code, since Colored and Flavored do not exist as individual concepts.

⁶ Combining the ones used in those works [34, 38], with abstract methods to allow self instantiation as in [41].

We also compare with a solution in Scala, offering the same level of reuse and code modularity of the 42_{μ} solution, but again it is more verbose and requires more members (31): an indication that it may be logically heavier too. We define the main tPoint trait (8 lines), the tPointSum operation (3), the two tColored and tFlavored traits (6 * 2) and the CPoint and CFPoint classes (12 + 18). The major benefit of 42_{μ} is the reduction of the amount of glue-code needed to generate CPoint and CFPoint (4 + 9).

The results for the second experiment are presented below.						
Language	Lines of code	N. of members	N. of classes or traits			
Java	47 = 10 + 9 + 13 + 15	27	6			
Scala	53 = 8 + 3 + 6 * 2 + 12 + 18	31	6			
42_{μ}	33 = 7 + 3 + 5 * 2 + 4 + 9	24	6			

The results for the second experiment are presented below.

5 Family Polymorphism by Disconnecting Use and Reuse

A nested class is just another kind of member in a code literal. In Java and Scala if a subclass declares a nested class with the same name of a nested class of a superclass, the parent declaration is simply hidden. The main idea of family polymorphism (FP) [12, 16, 21, 24, 25, 32, 36] is to instead consider such definition a form of *overriding*, called *further extension*. That is, the following Java code is ill typed:

```
1 abstract class A{static class B{..} abstract B m();}
```

```
2 class AA extends A{static class B{..} B m(){..}}//Error: Invalid overriding
```

```
3 //method AA.m() return type is AA.B, that is unrelated to A.B
```

In the FP approach, class AA.B would further extend A.B, thus the overriding of method A.m would be accepted. We extend 42_{μ} with nested classes, so that by composing code with **Use**, nested classes with the same name are recursively composed. The corresponding code in 42_{μ} would work, and behave like further extension in FP.

```
1 a={B={..} method B m()}
2 AA= Use a, {B={..} method B m(){..}}
```

For simplicity, we discuss nested classes but not nested traits: and all traits and code composition expressions are still at top level. In this way all dependencies are about top level names, allowing the type system to consider the class table as a simple map from (nested) type names (such as A and A.B.C) to their definition.

There are a lots of different forms of rename in literature [1, 10, 16]. Here we introduce a simple variant to rename nested types to other nested types. For example:

```
1 t={ method B m() B={ method B mb()} }
2 D= t[rename B into C]
```

would flatten to:

```
1 t={ method B m() B={ method B mb()} }
2 D={ method C m() C={ method C mb()} }
```

The rename only influences its argument. Since traits do not induce nominal types, we can consistently change their internally used names without breaking any code. The full L42 offers many other kinds of renames, but we do not need them to show our next example.

Application to the expression problem. Case Study 3:

The above extensions lets us challenge the expression problem [39], with the requirements exposed in [41]. In the expression problem we have data-variants and operations and we can *extend our solution in both dimensions*, by adding new datavariants and operations. We aim to *combine independently developed extensions* so that they can be used jointly. To be modular, extensions will preserve *type safety* and allow *separate compilation* (no re-type-checking), while avoiding *duplication of source code*.

Following closely the example of Zenger and Odersky [41], we consider a language where the expressions Exp can be Num (for number literal), Plus (for binary plus operator) and Neg (for unary minus). We then proceed to define operations show to convert them into strings, eval to compute their numeric values and double to double their containing Nums. We thus have 3 classes, 1 interface, the definition of the state, and 3 operations. We model this as a table of features, as in [16]: a (3 classes + 1 interface)*(1 state + 3 operations) table composed by 16 traits. The features are atomic: they exactly declare the state of a class or define a single operation for a single class. 42_{μ} avoids the large amount of abstract declarations that clutters the solution in [16]. Intuitively, we would like our traits to look like the following:

```
1 evalPlus= Use plus, {//eval operation for Plus data-variant
2 Exp= {interface
3 method int eval()}
4 Plus= {implements Exp
5 method int eval(){
6 return this.left().eval()+this.right().eval();}}
```

evalPlus uses the trait plus to import the state (the left() and right() methods) and defines the eval() method from interface Exp. But, if we were to declare those explicitly, we would repeat Exp, the abstract declaration of eval() and 'implements Exp' for all data-variants. To avoid this duplication, we write the trait eval with a placeholder T nested class, that can then be renamed into the corresponding data-variant. Thus, our source code is as follows; First we declare the 4 traits to represent the state:

1 exp= { Exp= {interface} T= {implements Exp}}

```
num= Use exp[rename T into Num],{//T is renamed to Num and summed with
Num= {method int value() static method Num of(int value)}} // this Num
plus= Use exp[rename T into Plus], {
Plus= {method Exp left() method Exp right()
static method Plus of(Exp left,Exp right)}}
neg= Use exp[rename T into Neg],{
Neg= {method Exp term() static method Neg of(Exp term)}}
```

Here we define a trait for each data-variant. Each trait will contain its version of Exp and a specific kind of expression, with its state. Next, we define the operation eval for all the data-variants. The former solutions in [16] required repeating the state declaration of the data-variant in each operation, while we can just import it.

1 eval= {Exp= {interface method int eval()} T= {implements Exp}}

1 evalNum= Use num, eval[rename T into Num],{ //just the implementation

2 Num= { method int eval(){return this.value();} }}//of the specific method

```
1 evalPlus= Use plus, eval[rename T into Plus], {
2   Plus= { method int eval(){ return this.left().eval()+this.right().eval();} }}
```

```
1 evalNeg= Use neg, eval[rename T into Neg], {Neg={ method int eval(){..}}}
```

The show operation can be trivially defined following exactly the same pattern (omitted here for space reasons). The operation double is a challenge for some proposed solutions to the expression problem, as explained by Zhang and Oliveira [42]. The double operation is called a *transformation*: an operation from Exp to Exp. Thanks to 42_{μ} 's separation between use and reuse, together with support for self-instantiation of nested classes double does not need any special attention and can be written just like eval and show.

```
1 double= { Exp= {interface method Exp double()} T= {implements Exp} }
1 doubleNum= Use num, double[rename T in Num],{
2 Num= { method Exp double(){return Num.of(this.value()*2);} }}
1 doublePlus= Use plus,double[rename T in Plus],{
2 Plus= { method Exp double(){
3 return Plus.of(this.left().double(),this.right().double());} }}
1 doubleNeg=....
```

Here we define a trait for each data-variant implementing the operation double(). Again, each trait will contain its version of Exp with double() and a specific kind of expression, with the implementation for double() for that specific kind.

Our third case study compares with the results presented in Scala [41]. The proposed solution is not fully modularized as a table, so in order to make a more close comparison, we provide an alternative version where we isolate all the units of behaviour as is done in 42_{μ} .

lines	methods	Coole was 10 4 v 2 methods
52	15 = 12 + 3	Scala uses $12 = 4 \times 3$ methods
78	15 = 12 + 3	plus 3 extra factory methods (for
27	3	double). We use $12 = 4 \times 3$ meth-
48	$19 = 4 \times 3 + 7$	ods plus our abstract state: 4 get-
3	0	ters and 3 factories.
	lines 52 78 27 48 3	linesmethods 52 $15 = 12 + 3$ 78 $15 = 12 + 3$ 27 3 48 $19 = 4 \times 3 + 7$ 3 0

As we can see, encoding atomic units in Scala is more verbose, but more importantly, in 42_{μ} we can just define a class supporting any subset of operations and data-variants by listing the desired traits: for example, a solution for Num and Plus (but not Neg) with eval and double would look like this: Example= **Use** evalNum, evalPlus, doubleNum, doublePlus. The composition of all our traits would just requiring listing all of the relevant behaviour; reasonably formatted, it could take up to 3 lines. On the other hand, the presented solution in Scala requires 27 lines of glue code to put the traits together. This means that a full Scala solution *requiring a single instantiation with all the traits* would be 78+27 = 105 lines. If we were to require more instantiations with a different subset of traits, the glue code would dominate the line count, and the Scala solution would end up being up to 9 times heavier than the 42_{μ} one (if all 64 permutations were required).

The line count for 42_{μ} is very predictable: after defining exp (3) and the state traits (4+6+5) for each of the three operations (eval, show, double) we just needed 4 lines to declare the operation in the interface, and 2 lines for each of the 3 data-variants.

Following [41], after double we present an implementation of equals. Their solution involved double dispatch to avoid casting. To show understandable code, we show a simpler solution with a guarded cast (sometime called a typecase).⁷ The idea is that since every data-variant contains the same "cast" logic, we can modularize it into an equals trait; equals in [41] is complex and and requires glue code.

```
1 equals= {
  Exp= {interface
                        method Bool equals(Exp that)}
2
  T= {implements Exp
    method Bool exactEquals(T that)
4
5
    method equals(that){
      if(T instanceof This){return this.exactEquals(that);}else{ return false;}}}
6
1 equalsNum= Use num, equals[rename T into Num],{
   Num= {method Bool exactEquals(Num that){
2
    return this.value().equals(that.value());} }}
3
1 equalsPlus= Use plus, equals[rename T into Plus],{
   Plus= {method Bool exactEquals(Plus that){
2
    return this.left().equals(that.left()) && this.right().equals(that.right());
4
   } }}
```

```
1 equalsNeg= Use neg, equals[rename T into Neg],{
2 Neg= {method Bool exactEquals(Neg that){
3 return this.term().equals(that.term());} }}
```

	lines	methods
Original Scala equals	40	10
Isolated Scala equals	31	10
Scala equals instance	29	3
42 trait eq d-dispatch	21	6
42 class dd instance	22	11
42 traits eq Cast	13	6
42 class cast instance	3	0

The Scala code here can be made fully isolated with little extra syntactic cost. The original Scala eq is 40 lines and contains a part of the glue code mixed inside. The isolated version is 31 lines and to merge all the operations together in Scala, it takes 29 lines of glue code. Note that this is mostly the same glue code from before (27 lines), that needs to be manually adapted.

In 42_{μ} we are more compact than Scala both when using the double dispatch (21 + 22 vs. 31 + 29) or the guarded cast (13 + 3 vs. 31 + 29). To instantiate the double dispatch version in 42_{μ} we need 22 lines of glue code. We could remove such glue code using features from the full 42 language, but here we stick to only the features presented in this paper. The interesting point is that the nature of our needed glue code is different with respect to the Scala glue code: Scala requires lots of trait multiple inheritance declarations to explicitly merge nested traits with the same name, while in 42_{μ} we mostly need to add the negative cases for the double dispatch (such as Sum={method Bool equalToNum(Num that){return false;}}).

⁷ The interested reader can find a 42_{μ} implementation of equals with double dispatch in the appendix.

6 Summary of formalisation

In Appendix A we formalise 42_{μ} ; in addition to conventional soundness, we discuss detailed behaviour and soundness of the compilation process itself; a similar property was called meta-level-soundness in [37]. This property ensures that flattening strictly reduces the number of type errors. In turn, this ensure that reusing a trait cannot induce new type errors. This property was already proved in [37]; here the proof is smoother thanks to our simpler formalisation. Our process requires traits to be well-typed before being reused, however code literals are not required to be well typed before flattening. This design supports mutually recursive types without having to predict the structure of the flattened code, as was needed in [16].

7 Related Work

Literature on code reuse is too vast to let us do justice to it in a few pages. In particular, we were unable to discuss all the variations on family polymorphism. Our work is inspired by traits [18], which in turn are inspired by module composition languages [1].

7.1 Separating Inheritance and Subtyping

In languages like Cecil [14] and PolyToil [13], classes are not types: it is a more radical solution to 'inheritance implies subtyping', and equivalent to a restricted version of 42_{μ} where only interface names can be used as types. This complicates typing of **this**, and may prevent any useful application of the **This** type (PolyToil uses polymorphism to support it). Those approaches would ban the following code, since A is not a type:

1 class A{ int ma(){return Utils.m(this);} } class Utils{static int m(A a){..}}

Cecil syntactic sugar counters this issue; advantages and pitfalls are discussed in Appendix B (especially B.2).

 42_{μ} is directly inspired by the 3 independently designed research languages as already mentioned: TraitRecordJ (TR)[6], Package Templates (PT)[26] and Deep-FJig(DJ) [16]. We synthesize the best ideas of those very different designs, while at the same time coming up with a simpler and improved design for separating subclassing from subtyping, which also addresses various limitations of those 3 particular language designs. TR, DJ, and PT are research projects, aiming to be platforms to experiment concepts, not to expose a compact syntax to programmers; instead of using case studies to compare 42_{μ} against TR, DJ, and PT, in the following we compare various aspects of the language designs on a more theoretical level. We **bold** 3 properties where one approach shines the most, and we squiggle 3 properties where one approach is more lacking.

A simple uniform syntax for code literals. Between those tree approaches, DJ is best in this sense: TR has separate syntax for class literals, trait literals and record literals. PT is built on top of Java, thus, it must support many different syntactic forms. 42_μ relies on DJ's approach but, thanks to our novel representation of state, 42_μ offers a much simpler and uniform syntax than DJ, TR, and PT.

- Reusable code cannot be "used" (that is be instantiated or used as a type). This happens in TR and in PT, but not in DJ. In DJ, to allow reusable code to be directly usable, classes introduce nominal types in an unnatural way: the type of this is only This (sometimes called <>) and not the nominal type of its class. That is in DJ 'A={method A m(){return this;}}' is not well typed. This is because 'B= Use A' flattens to 'B={method A m(){return this;}}', which is clearly not well typed. Looking at this examples makes it clear why we need reusable code to be agnostic of its name as in TR, PT, and 42_µ: either reusable code does not correspond to a type name (as in TR, PT, and 42_µ) or all code is reusable and usable, and all code needs to be awkwardly agnostic of its name, as in DJ.
- Requiring abstract signatures is a left over of module composition mindset. TR and DJ comes from a tradition of functional module composition, where modules are typed in isolation under an environment, and then the composition is performed. As we show in this work, this ends up requiring verbose repetition of abstract signatures which (for highly modularized code) may end up constituting most of the program. Java (and thus **PT**, as a Java extension) show us a better way: names are understood from their reuse context. The typing of PT offers the same advantages of the 42_{μ} typing model, but is more indirect. This may be caused by the heavy task of integrating with full Java. Recent work based on TR is trying to address this issue too [17].
- Composition algebra. The idea of using composition operators over atomic values as in an arithmetic expression is very powerful, and makes it easy to extend languages with more operators. 42_μ, DJ, and TR embrace this idea, while PT takes the traditional Java/C++ approach of using an enhanced class/package declaration syntax. The typing strategy of PT also seems to be connected with this decision, so it would be hard to move their approach to a composition algebra setting.
- Complete ontological separation between use and reuse. While 42_μ, TR, DJ, and PT all allow separating inheritance and subtyping only 42_μ and TR properly enforce separation between use (classes and interfaces) and reuse (traits). In DJ all classes are both units of use and reuse (however, subtyping is not induced). PT imports all the complexity of Java: it is possible to separate use and reuse, the model has powerful but non-obvious implications where Java extends and PT are used together.
- Naming the self type, even if there is none yet. TR is lacking here, while 42_μ, DJ and PT both allow a class to refer to its name; albeit this is less obvious in PT since both a package and a class have to be introduced to express it. This allows encoding binary methods, expressing patterns like withers or fluent setters and to instantiate instances of (future) classes using the reused code.

7.2 Implications for Family Polymorphism

Our **Use** operator is similar to deep mixin composition [19, 22, 41] and family polymorphism [21, 24, 25, 36], but is symmetric while the operator **super** offers flexible explicit conflict resolution. Our presented solution to the expression problem improves over existing solutions in the literature, where one close contender is DJ [16]: our gain over their model is based on our relaxation over abstract signatures. A similar syntax can be achieved with the Scandinavian style [20], or with the work of Nystrom (Jx [31] and J& [32]), where the composition behaves similarly to our sum operator. Both Jx, J&, and the virtual classes of Ernst [21] make use of dependent types. As in .FJ and ^FJ [24, 25, 34, 36], we do not need sophisticated types. The work on DJ [16] contains an in-depth comparison between various FP approaches, including an example written in .FJ syntax synthesizing the difficulty of supporting FP while keeping Use and Reuse connected:

```
1 class A{static class B{int f1;} int k(.B x){ return x.f1;}}
```

```
2 class AA extends A{static class B{int f2;} int k(.B x){return x.f2+new .B().f2;}}
```

The syntax .B denotes a relative path, that is, the class B in scope. In FP AA.B further extends A.B: it is implicitly considered a subclass of A.B, adding the field f2. Consider now the following code:

```
1 new AA().k(new AA.B())//well-typed
```

```
2 new A().k(new A.B())//well-typed
```

```
3 A a=new AA(); //well-typed assuming AA is a subtype of A
```

```
4 a.k(new A.B())//runtime error: A.B.f2 does not exist
```

In the sound .FJ type system the last method invocation illtyped even though AA.B is a subtype of A.B. With minor changes, others [12, 21, 25, 32] support this example in the same way. Inheritance implying subtyping is broken only in a controlled way, and it is allowed whenever it does not lead to unsoundness. Recent work on **ThisType** [33, 35] also continues in this line. In those works, "subtyping by subclassing" is preserved: those designs aim to retain the programming model of mainstream OOP languages and backwards compatibility. 42 is instead a radical departure from mainstream OOP, hoping to improve the mechanisms for use and reuse in OOP and unlock new ways to design software.

From a different perspective, we can say that traditional implementations of family polymorphism are still heavily influenced by the "inheritance implies subtyping" model. We believe that this is a major source of complexity in the type systems of those approaches: they need to track calls, and enforce that the *family* of the receiver and the argument is the same. Because we separate inheritance from subtyping we liberate ourselves from tricky issues that arise in such type systems, and can provide a simpler model of family polymorphism, soundly supported by a straightforward nominal type system: by disconnecting use and reuse we outlaw A a=new AA(). In 42_{μ} this also reduces the expressive power a little, but in the full 42 language, as well as in DJ, the operator *redirect* allows us to write code that is parametric on families of data types. To the same aim, .FJ relies on generics.

Support for FP strictly includes support for the '**This**' type and self instantiation. Scala allows *encoding* further extension/deep mixin composition, but it requires doing it explicitly, growing the amount of required glue-code.

7.3 State and traits

The original trait model [18] has no self instantiation and avoids any connection between state and traits. Since it was applied to a dynamic language, the relation with the '**This**' type is unclear.

The idea of abstract state operations emerged from Classless Java [40]. This approach offers a clean solution to handle state in a trait composition setting. Note how abstract state operations are different from just hiding fields under getters and setters: in our model the programmer simply never has to declare what information is stored in fields. The state is computed by the system as an overall result of the whole code composition process.

In the literature there have been many attempts to add state in traits and in module composition languages:

- The simplest solutions have no constructors, and fields start with null (or zero/false). In this setting fields are another kind of (abstract) member, and two fields with identical types can be merged by sum/use; new C() can be used for all classes, and init methods may be called later, as in Point p=new Point(); p.init(10,30). This approach is commutative and associative. However, objects are created "broken" and the user is trusted with fixing them. While it is easy to add fields, the load of initializing them is on the user; moreover all the objects are intrinsically mutable,
- **Constructors compose fields**: Here a canonical constructor (as in FJ) taking a parameter for each field and just initializing the fields is assumed to be present. It is easy to add fields, however this model (used by [28]) is associative but not commutative: composition order influences field order, and thus the constructor signature. Self instantiation is also not possible since the signature of the constructors change during composition.
- Constructors can be composed if they offer the same exact parameters: In this model, traits declare fields and constructors initialize their fields using any kind of computation. Traits whose constructors have the same signature can be composed. The composed constructor will execute both constructor bodies in order. This approach is designed in DJ to allow self instantiation. It is associative and mostly commutative: composition order only influences execution order of side effects during construction. However trait composition requires identical constructor signatures: this hampers reuse, and if a field is added, its initial value needs to be synthesized from the other parameters.

7.4 Tabular comparison of many approaches

preventing a functional programming style.

In this table we show if some constructs support certain features: direct instantiation (as in **new** C()), self instantiation (as in **new This**()), is this construct a 'unit of use'?, a 'unit of reuse'?, does using this construct introduce a type? and is the induced type the type of **this**?, support for binary methods, does inheritance of this construct induce subtyping?, is the code of this construct required to be well-typed before being inherited /imported to a new context? is it required to be well-typed before being

	direct instantiation	self instantiation	unit of use	unit of reuse	introduces a type	induced type is the type of this	binary methods	inheritance induces subtyping	typed before imported	typed before composed
Java/Scala classes	Y	Ν	Y	Y	Y	Y	Ν	Y	Y	Ν
Java8 interfaces	Ν	Ν	Ν	Y	Y	Y	Ν	Y	Y	Ν
Scala traits	Ν	Ν	Ν	Y	Y	Y	-	Y	Y	Ν
Original traits	Ν	Ν	Ν	Y	-	-	-	Ν	-	-
TR	Ν	Ν	Ν	Y	Ν	-	Ν	Ν	Y	Y
42_{μ} traits	Х	Y	Х	Y	Ν	-	Y	Ν	Y	Ν
42_{μ} classes	Y	Y	Y	Ν	Y	Y	Y	-	Y	-
Module composition	-	-	Y	Y	-	-	-	-	Y	Y
DJ classes	Y	Y	Y	Y	Y	Х	Y	Ν	Y	Y
PT	Ν	Y	Ν	Y	Ν	-	-	Ν	Y	Ν

composed with other code? Y and N means yes and no; we use "-" where the question is not applicable to the current approach. For example the original trait model was untyped, so typing questions make no sense there.

8 Conclusions, extensions and practical applications

In this paper we explained a simple model to radically decouple inheritance/code reuse and subtyping. Our decoupling does not make the language more complex: we replace the concept of abstract classes with the concept of traits, while keeping the concepts of interfaces and final classes. Concrete non final classes are simply not needed in our model. Thus, we believe that 42_{μ} is beneficial for code reuse in important cases without having negative impacts on the general programming experience. The interested reader can find a detailed discussion about this in Appendix B.

The model presented here is easy to extend. More composition operators can be added in addition to **Use**. Variants of the sophisticated operators of DJ are included in the full 42 language. Indeed we can add any operator respecting the following:

- When the operator fails it needs to provide an error that will be reported to the programmer.
- When only well typed code is taken in input, if a result is produced, such result is also well typed.
- When the result is not well typed, the type error must be traced back to a fault in the input.

Our simplified model represents the conceptual core of 42: a novel full blown programming language. In full 42 code literals are first class values, thus we do not need explicit names for traits: they are encoded as methods returning a code literal.

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A Formalisation

Here we show a simple formalization for the language we presented so far. We also model nested classes, but in order to avoid uninteresting complexities, we assume that all type names are fully qualified from top level, so the examples shown before should be written like: This.Exp, This.Sum, etcetera. In a real language, a simple pre-processor may take care of this step.

In most languages, when implementing an interface, the programmer may avoid repeating abstract methods they do not wish to implement, however to simplify our formalization, we consider source code always containing all the methods imported from interfaces. In a real language, a normalisation process may hide this abstraction.⁸ We also consider a binary operator sum (+) instead of the nary operator **use**. Figure 1 contains the complete formalization for 42_{μ} : syntax, compilation process, typing, and finally reduction.

A.1 Syntax

We use t and C to represent trait and class identifiers respectively. A trait (TD) or a class (CD) declaration can use either a code literal L, or a trait expression E. Note how in E you can refer to a trait by name. In full 42, we support various operators including the ones presented before and much more, but here we only show the single sum operator: +. This operation is a generalization to the case of nested classes of the simplest and most elegant trait composition operator [18]. Code literals L can be marked as interfaces. We use '?' to represent optional terms. Note that the interface keyword is inside curly brackets, so an uppercase name associated with an interface literal is a interface class, while a lowercase one is a interface trait. Then we have a set of implemented interfaces and a set of member declarations, which can be methods or nested classes. The members of a code literal are a set, thus their order is immaterial. If a code literal implements no interfaces, the concrete syntax omits the **implements** keyword.

Method declarations *MD* can be instance methods or **static** methods. A static method in 42_{μ} is similar to a **static** method in Java, but can be abstract. This is very useful in the context of code composition. To denote a method as abstract, instead of an explicit keyword we just omit the implementation *e*.

Finally, expressions *e* are just variables, instance method calls or static method calls. Having two different kinds of method calls is an artefact of our simplifications. In the full 42 language, type names are a kind of expression whose type helps to model metaclasses. Our values $v_{\overline{D}}$ are are just calls to abstract static methods: thanks to abstract state, we have no **new** expressions, but just factory calls. Thus values are parametric on the shape of the specific programs \overline{D} . We then show the evaluation context, the compilation context and full context.

⁸ In the full 42 language scoping is indeed supported by an initial de-sugaring, and a normalisation phase takes care of importing methods from interfaces.

$ID ::= t \mid C$	class or trait name
DE ::= ID = E	Meta-declaration
D ::= ID = L	Declaration
$E ::= L \mid t \mid E + E \mid \dots$	Code Expression
$L ::= \{ interface? implements-T \overline{M} \}$	Code Literal
$T ::= C \mid C.T$	Туре
M ::= static? method $T m(\overline{Tx}) e? C = L$	Member
$e ::= x \mid e.m(\overline{e}) \mid T.m(\overline{e})$	Expression
$v_{\overline{D}}$::= $T.m(\overline{v_{\overline{D}}})$, where <i>m</i> is abstract in $\overline{D}(T)$	value
$\mathscr{E}_{\overline{D}} ::= [] \mathscr{E}_{\overline{D}}.m(\overline{e}) v_{\overline{D}}.m(\overline{v_{\overline{D}}},\mathscr{E}_{\overline{D}},\overline{e}) T.m(\overline{v_{\overline{D}}},\mathscr{E}_{\overline{D}},\overline{e})$	evaluation context
$\mathscr{E}_c ::= [] \mid \mathscr{E}_c + E \mid L + \mathscr{E}_c \mid \dots$	compilation context
$\mathscr{E} ::= [] \mid \mathscr{E} * E \mid E * \mathscr{E} \mid \dots$	ctx
$\Gamma ::= x_1:T_1,\ldots,x_n:T_n$	variable environment
$F_{0} \rightarrow F_{1} \qquad \forall D \in \overline{D} \ \overline{D} \vdash D : OK \qquad (LOOK-UP) \qquad (CIX-C)$	$\rightarrow F_1$ (SUM)
	$\underbrace{\overline{D}}^{T} \underbrace{L_{1}}_{L_{1}} \underbrace{L_{1}}_{L_{1}} \underbrace{L_{2}}_{L_{1}} \underbrace{L_{1}}_{L_{2}} \underbrace{L_{2}}_{L_{1}} \underbrace{L_{2}} \underbrace{L_{2}}_{L_{1}} \underbrace{L_{2}}_{L_{1}} \underbrace{L_{2}} \underbrace{L_{2}} \underbrace{L_{2}}_{L_{1}} \underbrace{L_{2}} $
$\overline{D} \ \overline{D'} \ ID = E_0 \ \overline{DE} \to \overline{D} \ \overline{D'} \ ID = E_1 \ \overline{DE} \qquad t \xrightarrow{\longrightarrow} \overline{D}(t) \qquad \mathscr{E}_c[E_0]$	$\xrightarrow{\overline{p}} \mathscr{E}_{c}[E_{1}] L_{1} + L_{2} \xrightarrow{\overline{p}} L \qquad \stackrel{1}{\longrightarrow} 2$
(CD-OK) (TD-OK)	
$C; \overline{D}, C=L_1 \vdash L_1 : \text{OK } L_1 = L_0[\text{This} = C]$ This; \overline{D} , This	$=L \vdash L : OK $ (s-refl)
$\overline{D} \vdash C = L_0 : OK coherent(C, L_1) \overline{D} \vdash t = L$	$L : OK$ $\overline{\overline{D}} \vdash T \le T$
(L-OK) (Nested-OK) (sue	STYPE)
$\forall M \in \overline{M}, T; \overline{D} \vdash M : \text{OK} \qquad T.C; \overline{D} \vdash L : \text{OK} \qquad \overline{D} \vdash T$	$T_2 \leq T_3 \ \overline{D}(T_1) = \{ \text{ implements } \overline{T} \}$
$\overline{T;\overline{D}} \vdash \{ _ \texttt{implements} \ \overline{T} \ \overline{M} \} : OK \overline{T;\overline{D}} \vdash C = L : OK \overline{\overline{D}} \vdash C = L : OK \overline{D} \vdash C = L : OK \overline{\overline{D}} \vdash C = L : OK \overline{\overline{D} \vdash C = $	$\overline{T_1 \le T_3} T_2 \in \overline{T}$
(Method-OK)	
if e ? = e then \overline{D} ; $\Gamma \vdash e : T_0$ if stat	tic? = static
$\overline{T;\overline{D}} \vdash \text{static? method } T_0 \ m(T_1 x_1 \dots T_n x_n) \ e? : OK$ the else	$ \Gamma = x_1 : T_1 \dots x_n : T_n $ $ \Gamma = \texttt{this} : T, x_1 : T_1 \dots x_n : T_n $
(SUBSUMPTION)	
\overline{D} ; $\Gamma \vdash e: T_1$ (STATIC-METHOD-CALL)	
$\overline{D} \vdash T_1 \leq T_2$ $\overline{D}; \Gamma \vdash e_1 : T_1 \dots \overline{D}; \Gamma \vdash e_n : T_n$	_
$\frac{1}{\overline{D}:\Gamma \vdash a:T} \qquad \frac{\overline{D}:\Gamma \vdash T}{\overline{D}:\Gamma \vdash T} m(a = a):T$ static m	$\texttt{nethod} \ T \ m(T_1 x_1 \dots T_n x_n) _ \in D(T_0)$
$D, 1 + e \cdot 1_2$ $D, 1 + 1_0 \cdot m(e_1 \cdot \dots \cdot e_n) \cdot 1$	
(X) (METHOD-CALL)	
$\overline{D}; \Gamma \vdash e_0 : T_0 \dots \overline{D}; \Gamma \vdash e_n : T_n$	$(T, r, T, r, r) \in \overline{D}(T)$
$\frac{1}{D \cdot \Gamma \vdash v \cdot \Gamma(v)} = \frac{1}{D \cdot \Gamma \vdash a m(a - a) \cdot T} \text{method } I \Pi$	$[(I_1 x_1 \dots I_n x_n)] \in D(I_0)$
$D, 1 \vdash x \cdot 1(x)$ $D, 1 \vdash e_0 \cdot m(e_1 \cdot \cdot e_n) : 1$	
(CTXV) (S-M) (M)	
$ \begin{array}{ccc} \nu, \mathbf{i} \vdash x \cdot \mathbf{i}(x) & \nu; \mathbf{i} \vdash e_0.m(e_1 \dots e_n): \mathbf{i} \\ (\text{CTXV}) & (\text{s-M}) & (\text{M}) \\ e_0 \xrightarrow{\nu}{\mathbf{p}} e_1 & (\text{M}) \end{array} $	
$\begin{array}{ccc} D, \Gamma \vdash x \cdot \Gamma(x) & D, \Gamma \vdash e_0.m(e_1 \dots e_n) : I \\ (CTXV) & (S-M) \\ \hline e_0 \xrightarrow{\overline{D}} e_1 \\ \hline \mathcal{E}_{\overline{D}}[e_0] \xrightarrow{\overline{D}} \mathcal{E}_{\overline{D}}[e_1] & \hline T.m(\overline{\nu_{\overline{D}}}) \xrightarrow{\overline{D}} meth(\overline{D}(T,m),\overline{\nu_{\overline{D}}}) & \hline \nu_{\overline{D}}.m(\overline{n_{\overline{D}}}) \end{array}$	$\overline{\overline{v_{\overline{D}}}}) \xrightarrow{\overline{D}} meth(\overline{D}(T,m), v_{\overline{D}} \overline{v_{\overline{D}}}) v_{\overline{D}} = T.m'(_)$

Figure 1 Formalization

A.2 Well-formedness

The whole program (\overline{DE}) is well formed if all the traits and classes at top level have unique names. The special class name **This** is not one of those, and the subtype relations are consistent: this means that the implementation of interfaces is not circular, and that $\forall ID = \mathscr{E}[L] \in \overline{DE}$, consistentSubtype(\overline{DE} , **This**=L;L). That is, every literal declares all the methods declared in its super interfaces. The full 42 language allows covariant return types as in Java. Here for simplicity we require them to have the same type declared in the super interface.

Define consistentSubtype(\overline{DE} ;L)

• consistentSubtype(DE, { interface?implements $\overline{T} \ \overline{M}$ }) where $\forall T \in \overline{T}, \quad \overline{DE}(T) = \{ interface_{} \}, ^{9}$ $\forall _=L \in \overline{M}, \quad consistentSubtype(\overline{DE}; L) \text{ and}$ $\forall m, T \in \overline{T}, \quad \text{if method } T_{0} \ m(\overline{Tx}) \in \overline{DE}(T) \text{ then method } T_{0}m(\overline{Tx}) \ e? \in \overline{M}$

A code literal *L* is well formed iff:

- for all methods: parameters have unique names and no parameter is named this,
- all methods in a code literal have unique names,
- all nested classes in a code literal have unique names, and no nested class is called This,
- all used variables are in scope, and
- all methods in an interface are abstract, and they contain no static methods.

A.3 Compilation process

The compilation process is particularly interesting, it includes the flattening process and how and when compilation errors may arise. It is composed by rules TOP, LOOK-UP, CTX-C and SUM. To model more composition operators, they would each need their own rule.

Rule TOP compiles the leftmost top level (trait or class) declaration that needs to be compiled. First it identifies the subset of the program \overline{D} that can already be typed (second premise). Then the expression is executed under the control of such compiled program (first premise). All the traits inside the expression need to be compiled (rule LOOK-UP): $\forall t$, if $E = \mathscr{E}[t]$ then $t \in \text{dom}(\overline{D})$. If the required \overline{D} cannot be typed, this would cause a compilation error at this stage. Rule LOOK-UP replaces a trait name twith the corresponding literal L. Since \overline{D} is all well typed, L is well typed too. Rule CTX-C uses the compilation context to apply a deterministic left to right call by value¹⁰ reduction; thus the leftmost invalid sum that is performed will be the one providing the compilation error.

⁹ That is, in this simplified version in order to implement an interface nested in a different top level name, such interface can not be generated using a trait expression. This limitation is lifted in the full language.

¹⁰ In the flattening process, values are code literals *L*.

Keeping in mind the order of members in a literal is immaterial, rule SUM applies the operator:

 $\begin{array}{l} \textbf{Define } L_1 + L_2, \ \overline{M} + \overline{M}, \ M + M \\ \bullet \ L_1 + L_2 = L_3 \quad \text{where} \\ L_1 = \{ \text{ interface? implements } \overline{T}_1 \ \overline{M}_1 \ \overline{M}_0 \} \\ L_2 = \{ \text{ interface? implements } \overline{T}_2 \ \overline{M}_2 \ \overline{M}_0' \} \\ L_3 = \{ \text{ interface? implements } \overline{T}_1, \overline{T}_2 \ \overline{M}_1, \overline{M}_2 \ (\overline{M}_0 + \overline{M}_0') \} \\ \text{ dom}(\overline{M}_1) \text{ disjoint } \text{ dom}(\overline{M}_2) \text{ and } \text{ dom}(\overline{M}_0) = \text{ dom}(\overline{M}_0') \\ \bullet \ (M_1 \dots M_n) + (M_1' \dots M_n') = (M_1 + M_1') \dots (M_n + M_n') \end{array}$

- $M_1 + M_2 = M_2 + M_1$
- $C=L_1 + C=L_2 = C=L_3$ if $L_1 + L_2 = L_3$
- static? method $T_0 m(\overline{Tx})$ + static? method $T_0 m(\overline{Tx})e$? = static? method $T_0 m(\overline{Tx})e$? Sum composes the content of the arguments by taking the union of their members and the union of their implements. Members with the same name are recursively composed. There are three cases where the composition is impossible.
- *Method-clash*: two methods with the same name are composed, but either their headers have different types or they are both implemented.
- *Class-clash*: a class is composed with an interface.¹¹
- Implements-clash: the resulting code would not be well formed. For example, in the following t1+t2 would result in a class B implementing A with method a(), but B does not have such method.¹²

```
1 t1={ A= {interface method Void a()} }
2 t2={ A= {interface} B= {implements A} }
```

Implements-clash can happen only when composing nested interfaces. Note that while the first two kind of errors are obtained directly by the definition of $L_1 + L_2$, Implements-clash is obtained from well-formedness, since injecting the resulting *L* in to the program would make it ill-formed by *consistentSubtype*(\overline{DE} , *L*).

A.4 Typing

Typing is composed by rules CD-OK, TD-OK, L-OK, NESTED-OK and METHOD-OK, followed by expression typing rules SUBSUMPTION, METHOD-CALL, X and STATIC-METHOD-CALL.

Rules CD-OK and TD-OK are interesting: a top level class is typed by replacing all occurrences of the name '**This**' with the class name C, and is required to be coherent. On the other hand, a top level trait is typed by temporarily adding a mapping for **This** to the typed program.

^{II} The full language relaxes this condition, for example an empty class can be seen as an empty interface during composition.

¹² In 42_{μ} it could be possible to try to patch class B, for example by adding an abstract method a(); we choose to instead give an error since in the full 42 language such patch would be able to turn coherent private nested classes into abstract (private) ones.

Define coherent(T,L)

• coherent(T,{interface? implements $\overline{T} \overline{M}$ }) holds where

 $\forall C = L' \in \overline{M}coherent(T.C,L')$

and either interface? = interface

or \forall method $T' m(\overline{Tx}) \in \overline{M}$, state(factory(T, \overline{M}), method $T' m(\overline{Tx})$)

A Literal is *coherent* if all the nested classes are coherent, and either the Literal is an interface, there are no static methods, or all the static methods are a valid *state* method of the candidate *factory*. Note, by asking for **method** $T' m(\overline{Tx}) \in \overline{M}$ we select only abstract methods.

Define factory (T, \overline{M})

• factory($T, M_1 \dots M_n$) = M_i = static method $Tm(_)$ where

 $\forall j \neq i$. M_j is not of the form static method __(_)

The factory is the only static abstract method, and its return type is the nominal type of our class.

Define state(M, M')

• state(static method $T m(T_1 x_1 \dots T_n x_n)$, method $T_i x_i()$)

• state(static method $T m(T_1 x_1 \dots T_n x_n)$, method T with $x_i(T_i$ that))

A non static method is part of the *abstract state* if it is a valid getter or wither. In this simple formalism without imperative features we do not offer setters.

Rule NESTED-OK helps to accumulate the type of **this** so that rule METHOD-OK can use it. Rule L-OK is so simple since all the checks related to correctly implementing interfaces are delegated to the well formedness criteria. The expression typing rules are straightforward and standard.

A.5 Formal properties

As can be expected, 42_{μ} ensures conventional soundness of expression reduction. This property is expressed on a completely flattened program (a program where all *E* are of form *L*):

Theorem A.1 (Main Soundness). *if* $\vdash \overline{D}$: *OK, e not of form* $v_{\overline{D}}$ *and* $\overline{D} \vdash e$: *T then e* $\leftarrow_{\overline{D}}$

The proof is standard since the flattened language is just a minor variation over FJ.

In addition to conventional soundness of expression reduction, 42_{μ} ensures soundness of the compilation process itself. A similar property was called meta-levelsoundness in [37]; here we can obtain the same result in a much simpler setting. We denote $wrong(\overline{D}, E)$ to be the number of *Ls* such that $E = \mathscr{E}[L]$ and not $\overline{D} \vdash L$: OK.

Theorem A.2 (Compilation Soundness). *if* $E_0 \xrightarrow{\overline{D}} E_1$ *then* $wrong(\overline{D}, E_0) \ge wrong(\overline{D}, E_1)$.

This can be proved by cases on the applied reduction rule:

- LOOK-UP preserves the number of wrong literals: $t \in \overline{D}$ and \overline{D} is well typed by TOP preconditions.
- SUM either preserves or reduces the number of wrong literals: the core of the proof is to show that the sum of two well typed literals produces a well typed one. A code literal is well typed (L-OK) if all its method bodies are correct. This holds since those same method bodies are well typed in a strictly weaker environment with

respect to the one used to type the result. This is because every member in the result of the sum is structurally a subtype of the corresponding members in the operands. Note that by well formedness, if sum is applied the result still respect *consistentSubtype*.

Compilation Soundness has two important corollaries:

- A class declared without literals is well-typed after flattening; no need of further checking.
- If a class is declared by using literals $L_1 ldots L_n$, and after successful flattening C = L can not be type-checked, then the issue was originally present in one of $L_1 ldots L_n$. This also means that as an optimization strategy we may remember what method bodies come from traits and what method bodies come from code literals, and only type-check the latter. If the result can not be type-checked, either it is intrinsically illtyped or a referred type is declared *after* the current class. As we see in the next section, we leverage on this to allow recursive types.

A.6 Advantages of our compilation process

Our typing discipline is very simple from a formal perspective, and is what distinguishes our approach from a simple minded code composition macros [4] or rigid module composition [1]. It is built on two core ideas:

1: Traits are *well-typed* **before being reused.** For example:

```
1 t={method int m(){return 2;}
```

```
2 method int n(){return this.m()+1;}}
```

t is well typed since m() is declared inside of t, while the following would be illtyped: t1={method int n(){return this.m()+1;}} //illtyped

2: Code literals are not required to be well-typed before flattening.

A literal *L* in a declaration *D* must be well formed and respect *consistentSubtype*, but it is not type-checked until flattening is complete: only the result is required to be well-typed. For example the following is correct since the result of the flattening is well-typed:

1 C= Use t, {method int k(){return this.n()+this.m();}}//correct code

The code literal {method int k(){ return this.n()+this.m();}} is not well typed: n, m are not locally defined. This code would fail in many similar works in literature [5, 7, 16] where the literals have to be *self contained*. In this case we would have been forced to declare abstract methods n and m, even if t already provides such methods.

This relaxation allows multiple declarations to be flattened one at the time, without typing them individually, and only typing them all together. In this way, we support

recursive types¹³ between multiple class declarations without the need of predicting the resulting shape.¹⁴

As seen in TOP, our compilation process proceeds in a top-down fashion, flattening one declaration at a time, a declaration needs to be type-checked where their type is first needed, that is, when they are required to type a trait used in a code expression. That is, in 42_{μ} typing and flattening are interleaved. We assume our compilation process stops as soon as an error arises. For example:

```
1 ta={method int ma(){return 2;}}
2 tc={method int mc(A a, B b){return b.mb(a);}}
3 A= Use ta
4 B= {method int mb(A a){return a.ma()+1;}}
5 C= Use tc, {method int hello(){return 1;}}
```

In this scenario, since we compile top down, we first need to generate A. To generate A, we need to use ta (but we do not need tc, in rule TOP, $\overline{D} = ta$ and $\overline{D}' = tc$). At this moment, tc cannot be compiled/checked alone: information about A and B is needed. To modularly ensure well-typedness, we only require ta to be well typed at this stage; if it is not a type-error will be raised immediately. Now, we need to generate c, and hence type-check tc. A is guaranteed to be already type-checked (since it is generated by an expression that does not contain any *L*), and B can be typed. Finally tc can be typed and reused. If the sum rule could not be performed (for example if tc had a method hello too) a composition error would be generated at this stage. On the other hand, if B and c were swapped, as in:

```
1 C= Use tc, {method int hello(){return 1;}}
2 B= {method int mb(A a){return a.ma()+1;}}
```

we would be unable to type tc, since we need to know the structure of A and B. A type error would be generated.

The cost: what expressive power we lose

We require declarations to be provided in the right dependency order, but sometimes no such order exists. An example of a "morally correct" program where no right order exists is the following:

```
1 t= { int mt(A a){return a.ma();}}
2 A= Use t, {int ma(){return 1;}}
```

Here the correctness of t depends on A, that is in turn generated using t. We believe any typing allowing such programs would be fragile with respect to code evolution, and could make human understanding of the code-reuse process much harder. In sharp contrast with others (TR, PT, DJ, but also Java, C#, and Scala) we chose to not support this kind of involved programs.

TR, PT, DJ, Java, C#, and Scala accept a great deal of complexity in order to predict the structural shape of the resulting code before doing the actual code reuse/adapta-

¹³ OO languages leverage on recursive types most of the times: for example String may offer a Int size() method, and Int may offer a String toString() method. This means that typing classes String and Int in isolation one at a time is not possible.

¹⁴ This is needed in full 42: it is impossible to predict the resulting shape since arbitrary code can run at compile time.

tion. Those approaches logically divide the program in groups of mutually dependent classes, where each group may depend on a number of other groups. This forms a direct acyclic graph of groups. To type a group, all depended groups are typed, then the signature/structural shape of all the classes of the group are extracted. Finally, with the information of the dependent groups and the current group, it is possible to type-check the implementation of each class in the group.

In the world of strongly typed languages we are tempted to first check that all will go well, and then perform the flattening. Such methodology would be redundant in our setting: we can only reuse code through trait names; but our point of relaxation is only the code literal: in no way can an error "move around" and be duplicated during the compilation process. That is, our approach allows safe libraries of traits and classes to be typechecked once, and then deployed and reused by multiple clients: as Theorem A.2 states, in 42_{μ} no type error will emerge from library code.

A.7 Expression reduction

Our reduction rules are incredibly simple and standard. A great advantage of our compilation model is that expressions are executed on a simple fully flattened program, where all the composition operators have been removed. From the point of view of expression reduction, 42_{μ} is a simple language of interfaces and final classes, where nested classes give structure to code but have no special semantics. The reduction of expressions is defined by rules CTX-V, S-M, and M. The only interesting point is the auxiliary function *meth*:

Define $meth(M, \overline{v_{\overline{D}}})$

- meth(static method T $m(T_1 x_1 \dots T_n x_n) e, v_{\overline{D}1} \dots v_{\overline{D}n}) = e[x_1 = v_{\overline{D}1}, \dots, e_n = v_{\overline{D}n}]$
- meth(method T m(T₁x₁...T_nx_n) e, $v_{\overline{p}0}, \ldots, v_{\overline{p}n}$) = e[this = $v_{\overline{p}0}, x_1 = v_{\overline{p}1}, \ldots, e_n = v_{\overline{p}n}$]
- meth(method $T_i x_i(), T.m(v_{\overline{D}1}...v_{\overline{D}n})) = v_{\overline{D}i}$
 - where $\overline{D}(T,m) =$ **static** method $Tm(T_1x_1...T_nx_n)$
- meth(method T with $x_i(T_i \text{ that}), T.m(v_{\overline{D}1} \dots v_{\overline{D}n}) v_{\overline{D}}) = T.m(v_{\overline{D}1} \dots v_{\overline{D}i-1}, v_{\overline{D}}, v_{\overline{D}i+1} \dots v_{\overline{D}n})$ where $\overline{D}(T,m) = \text{static method } Tm(T_1x_1 \dots T_nx_n)$

Here we take care of reading method bodies and preparing for execution. The first case is for static methods and the second is for instance methods. The third and fourth cases are more interesting, since they take care of the abstract state: the third case reduce getters and the fourth reduces withers. In our formalisation we are not modelling state mutation, so there is no case for setters.

We omit the proof of conventional soundness for the reduction. It is unsurprising, since the flattened calculus is like a simplified version of Featherweight Java [23].

B Benefits without heavy costs

In this section we discuss why we believe the design of 42_{μ} is not intrinsically harder to use than Java. This appendix is not fundamental to understanding the main content of the paper, but may clarify misconceptions.

B.1 Introducing more names

At first it may seem that our approach requires introducing and keeping more names in mind, as for the set and Set example; however this is not the case. The *user* of Set needs to keep in mind only the class Set. The *re-user* of set needs to keep in mind only the trait set. Those are quite different roles, so they may very well be handled by different programmers. As shown in the set/Set example, defining a reusable class takes only an extra line with respect to defining a final one. In Java the default is "non final", and the programmer can write **final** to prevent code reuse, while in 42 the default is reversed.

It could be possible to define syntax sugar, like this:

```
1 reusable Set={...}
2 //expands into
3 set={...}
```

```
4 Set=set
```

While this may sound appealing to Java programmers, it only saves one line, and it may obscure the actual behaviour of the code. This kind of behaviour is offered in Cecil [14], where it makes sense, since in Cecil classes are not types. Cecil offers syntax sugar to declare classes and interfaces with the same structural type in a single declaration.

B.2 Two ways to separate subclassing and subtyping

There are two different ways to separate subclassing and subtyping:

- (1) Classes are not types: in this solution (used by Cecil, Toil/PolyToil, and most structurally typed languages) classes and types are different concepts; thus sub-classing is not subtyping. This also means that every method call is dynamically dispatched. Some authors consider this to be a security problem, and it is out of those concerns that Java designers decided to support final classes.¹⁵
- (2) Classes are not extensible: this is our solution; classes are types, but can not be reused. This requires introducing another concept for code reuse: traits. We think our solution is much less radical than the former one. A key advantage is that all class types are exact types, thus method calls on them are statically dispatched; supporting easier reasoning and higher security and efficiency. On the other hand, interface types are not exact; obviously method calls on interface types are dynamically dispatched, since there is no implementation. This builds towards a simpler mental model, and prevents bugs where programmers may

¹⁵ For example, a Java programmer may assume the call myPoint.getX() to behave as specified in the Point class. However, dynamic class loading may be used in a malicious way to provide a Point instance with dangerous behaviour. This can easily be prevented by declaring Point **final**. To insist that all calls be dynamically dispatched requires disabling dynamic class loading or verifying all dynamically loaded code.

forget about dynamic dispatch and reasoning over method calls using the specific implementation found in the receivers static type.

Both ways enforce essentially the same solution for the 'this-leaking' problem:

```
1 class A{ int ma(){return Utils.m(this);} }
2 class Utils{static int m(A a){..}}
3 class B extends A{ int mb(){return this.ma();} }
```

In (I) the error is using A as a type in Utils.m, while in (2) the error is using **extends**. The solution proposed by (I) would look like this:

```
1 type IA{interface method int ma()}//type = interface
2 class Utils{static method int m(IA a){return ...} }
3 class A{implements IA
4 method int ma(){return Utils.m(this);}}
5 class B extends A{ int mb(){return this.ma();} }
```

while our solution (2) is just one line longer:

```
1 IA={interface method int ma()}//interface with abstract method
2 Utils={static method int m(IA a){return ...} }
3 ta={implements IA
4 method int ma(){return Utils.m(this);}}
5 A=Use ta
6 B=Use ta,{ int mb(){return this.ma();} }
```

B.3 Class hierarchies

By having all classes final, we prevent class hierarchies. However, we allow both trait hierarchies and interface hierarchies. We believe having both these hierarchies, one for code reuse, and one for subtyping, can improve code maintainability, since these two hierarchies will be able to evolve independently. Furthermore, having two hierarchies does not makes code comprehension harder, since as for before, they have different roles and thus are relevant only one at the time.

Often, class hierarchies are used as a hack in order to obtain code reuse (and in those cases, the LSP is often violated). In 42_{μ} , trait hierarchies will serve the same role better.

B.4 Constructors and initialization

Java allows constructors to perform arbitrary operations. The same behaviour can be obtained in 42_{μ} by using a factory method, like so:

```
1 class A{
2 public int x;
3 public A(String s){
4 this.x=/*any computation here*/;
5 }
6 }
```

could be expressed in 42_{μ} as

```
1 A={
2 method int x();
3 static method This of(int x)
4 static method This of(String s){
5 return This.of(/*any computation here*/);
6 }
7 }
```

For space reasons we have not yet explained how to allow traits to have their own state with a default initialization. This could be solved by adding a default initializer to the abstract state operations (getters, setters, withers). For example:

```
1 t={
2 method int x();
3 method void x(int x);
4 method int defaultX(){return 0;}//new concept here!
5 }
```

would loosely correspond to a Java abstract class:

```
1 abstract class T{
2 private int x=0;
3 public int x(){return x;}
4 public void x(int that){return this.x=that;}
5 }
```

and could be used to obtain concrete classes where the abstract factory does not need to initialize the field x. For example the following classes would be coherent:

```
1 A= Use t,{ static method This of()}
2 B= Use t,{ method String s(); static method This of(String s)}
3 ...
4 A a=A.of();
5 a.x()==0;//assert a.x()==0
6 a.x(42);
7 a.x()==42;//assert a.x()==42
```

Finally, when self instantiation is not needed, declaring the abstract factory can be made redundant, as is done in the full 42 language.

Moreover, when code reuse is involved, 42_{μ} is much simpler to use than Java, C++ or C#: a class extending another class usually needs to define a constructor calling a super constructor. This typically has as many parameters as the full set of fields. Those parameters are then used at least one time in the body of the constructor. In 42_{μ} , to compose traits without adding fields, there is no need to repeat the abstract factory. To add a field, you need to define a new abstract factory and implement the "super" factory method by calling the new one. Thus, when adding a field the amount of code between 42 and Java is about the same, but is much easier when no fields are added.

B.5 Self instantiation is very useful

In many cases the absence of self instantiation causes bugs: often programmers "think" that **new** C() inside of class C would create an instance of the "current class", while of course **class** D **extends** C still creates instances of C. The absence of self instantiation

also prevents using functional programming patterns in OO languages. Just looking at how tedious it is to implement an immutable Point class in Java with withers, it is sufficient to dissuade most programmers.

B.6 Comprehensibility and usability

Of course it could take a while to adapt from the Java model to the 42_{μ} one. However, when it comes to model complex reuse scenarios, we believe 42_{μ} is much more explicit and clear than Scala and other options.

In particular, with respect to Java/Scala we encourage more explicit code since:

- Sub-typing needs to be explicitly declared.
- When sub-typing is needed, an interface needs to be declared. Thus, when using a class type, it is always clear what implementation is referred to.
- 42_μ requires being explicit when subtyping and subclassing are both desired by requiring an extra interface.
- this is required to access fields/methods, so they can not be confused with local variables.
- Our runtime semantics, as in untyped languages, do not depend on any static type information.

The last point is quite important: languages where inheritance does not induce subtyping that were proposed in the past tend to have more complex object models, instead our flattened language is just a language of interfaces and final classes. Novice programmers may start by learning that. They could learn to use traits later, and define their own traits even later.

C Case studies complete code

Here, for reference, you can find the complete code of the case studies. It is quite long, we report it here to help reproducibility of our results. The interested reader can navigate it in order to gain a better understanding on how we encoded the various solutions in the various styles and languages, but is not needed to understand the overall value of our paper.

While evaluating our approach, we consider not only lines of code but also the number of methods and classes. Our particular examples are focused on code reuse, thus the method implementations are all trivial. We believe that in this particular context the number of methods/classes is a good indicator.

C.1 Point algebra

C.1.1 Java7

```
1 //case 0 of 16
2 class Point {//6 lines
3 final int x;
4 final int y;
5
6 public Point(int x, int y) {
```

```
7 this.x = x;
8 this.y = y;
8
9 }
10 }
11
12 // 1
13 class PointSum extends Point {//5*4
14 public PointSum(int x, int y) {
15 super(x, y);
15 super(x, y);
16 }
17
 18 public PointSum sum(Point that) {
    return new PointSum(this.x + that.x, this.y + that.y);
}
19
20
21 }
 22
23 // 2
24 class PointSub extends Point {
25 public PointSub(int x, int y)
26 super(x, y);
27 }
    public PointSub(int x, int y) {
2/ ;
28
29 public PointSub sub(Point that) {
30 return new PointSub(this.x - that.x, this.y - that.y);
31 }
33
34 // 3
35 class PointMul extends Point {
36 ctass PointMul extends Point {
36 public PointMul(int x, int y) {
37 super(x, y);

       super(x, y);
38 }
39
40 public PointMul mul(Point that) {
41 return new PointMul(this.x * that.x, this.y * that.y);
42 }
41
42
43
43
44
45 // 4
46 class PointDiv extends Point {
47 public PointDiv(int x, int y) {

    super(x, y);
}
48
49
50
51 public PointDiv div(Point that) {
     return new PointDiv(this.x / that.x, this.y / that.y);
}
52
53
54 }
55
56 // 5
57 class PointSumSub extends PointSum {//7*6
58 public PointSumSub(int x, int y) {
59 super(x, y);
60 }
61
62 public PointSumSub sum(Point that) {
    return new PointSumSub(this.x + that.x, this.y + that.y);
}
63
64
66 public PointSumSub sub(Point that) {
67 return new PointSumSub(this.x - that.x, this.y - that.y);
68 }
60 }
69 }
70
71 // 6
72 class PointSumMul extends PointSum {
73
74
    public PointSumMul(int x, int y) {
       super(x, y);
75
     }
76
     public PointSumMul sum(Point that) {
    return new PointSumMul(this.x + that.x, this.y + that.y);
}
77
78
79
80
81 public PointSumMul mul(Point that) {
82 return new PointSumMul(this.x * that.x, this.y * that.y);
83 }
84 }
85
86 // 7
87 class PointSumDiv extends PointSum {
    public PointSumDiv(int x, int y) {
    super(x, y);
}
88
89
90
91
93 return new PointSumDiv(this.x + that.x, this.y + that.y);
94 }
95
96
97
    public PointSumDiv div(Point that) {
       return new PointSumDiv(this.x / that.x, this.y / that.y);
```

```
98   }
99 }
100
101 // 8
102 class PointSubMul extends PointSub {
103 public PointSubMul(int x, int y) {
104 super(x, y):
        super(x, y);
105 }
106
107 public PointSubMul sub(Point that) {
108 return new PointSubMul(this.x - that.x, this.y - that.y);
109 }
108
110
111 public PointSubMul mul(Point that) {
        return new PointSubMul(this.x * that.x, this.y * that.y);
112
112
113 }
114 }
115
116 // 9
public PointSu
119 super(x, y);
120 }
121
121
122
public PointSubDiv sub(Point that) {
122 public PointSubDiv(this.x - that)

123 return new PointSubDiv(this.x - that.x, this.y - that.y);
124 }
125
126 public PointSubDiv div(Point that) {
127 return new PointSubDiv(this.x / that.x, this.y / that.y);
128 }
129 }
130
131 // 10
132 class PointMulDiv extends PointMul {
133 public PointMulDiv(int x, int y) {
134 super(x, y);
135 }
136
141 public PointMulDiv div(Point that) {
142 return new PointMulDiv(this.x / that.x, this.y / that.y);
143 }
144 }
145
146 // 11
147 class PointSumSubDiv extends PointSumSub {//9*4
148
     public PointSumSubDiv(int x, int y) {
149
        super(x, y);
150 }
153 return new PointSumSubDiv sum(Point that) {
154 }
155 }
155 }

public PointSumSubDiv sub(Point that) {
    return new PointSumSubDiv(this.x - that.x, this.y - that.y);
158 }
159
     public PointSumSubDiv div(Point that) {
    return new PointSumSubDiv(this.x / that.x, this.y / that.y);
}
160
161
162
163 }
164
165 // 12
166 class PointSumSubMul extends PointSumSub {
167 public PointSumSubMul(int x, int y) {
168
168
        super(x, y);
     }
169
170
171 public PointSumSubMul sum(Point that) {
172 return new PointSumSubMul(this.x + that.x, this.y + that.y);
173 }
174
176 return new PointSumSubMul(this.x - that.x, this.y - that.y);
177 }
175 public PointSumSubMul sub(Point that) {
178
     return new PointSumSubMul(this.x * that.x, this.y * that.y);
}
179 public PointSumSubMul mul(Point that) {
180
181
182 }
 183
184 // 13
185 7/ 13
185 Class PointSumMulDiv extends PointMulDiv {
186 public PointSumMulDiv(int x, int y) {
197
187 super(x, y);
188 }
```

```
189
 190
                public PointSumMulDiv sum(Point that) {
 191
                     return new PointSumMulDiv(this.x + that.x, this.y + that.y);
               }
 192
 193
               public PointSumMulDiv mul(Point that) {
 194
 195
                    return new PointSumMulDiv(this.x * that.x, this.y * that.y);
              }
 196
 197
 198
               return new PointSumMulDiv(this.x / that.x, this.y / that.y);
}
               public PointSumMulDiv div(Point that) {
 199
 200
 201 }
 202
 203 // 14
203 // 14
204 class PointSubMulDiv extends PointMulDiv {
205 public PointSubMulDiv(int x, int y) {
206 class PointSubMulDiv(int x, int y) {
207 class PointSubMulDiv(int x, int y) {
208 class PointSubMulDiv(int x, int y) {
209 class PointSubMulDiv(int x, int x) {
209 class PointSubMulDiv(int x) {
209 class PointSubMulDiv
             super(x, y);
}
 206
 207
209 public PointSubMulDiv sub(Point that) {
210 return new PointSubMulDiv(this.x - that.x, this.y - that.y);
211 }
 208
212
213 public PointSubMulDiv mul(Point that) {
213 public PointSubMulDiv(this.x * tha

return new PointSubMulDiv(this.x * that.x, this.y * that.y);
215 }
 216
 217 public PointSubMulDiv div(Point that) {
218 return new PointSubMulDiv(this.x / that.x, this.y / that.y);
219 }
220 }
221
 222 // 15
 223 class PointSumSubMulDiv extends PointSumSubMul {//11
225 super(x, y);
226 }
 224 public PointSumSubMulDiv(int x, int y) {
 227
public PointSumSubMulDiv sum(Point that) {
    return new PointSumSubMulDiv(this.x + that.x, this.y + that.y);
}
 231
232 public PointSumSubMulDiv sub(Point that) {
233 return new PointSumSubMulDiv(this.x - that.x, this.y - that.y);
234 }
 235
 236 public PointSumSubMulDiv mul(Point that) {
237 return new PointSumSubMulDiv(this.x * that.x, this.y * that.y);
238 }
 239
 240 public PointSumSubMulDiv div(Point that) {
241 return new PointSumSubMulDiv(this.x / that.x, this.y / that.y);
242 }
 243 }
```

C.1.2 Classless Java

```
1 // 0
 2 @Obj // 3 lines
3 interface Point
    static Point of(int x, int y) { // "of" methods are generated by @Obj thus their lines are not counted
return new Point() {
 4
 5
        public int x() {
  return x;
 6
       3
 8
 9
10
        public int y() {
        return y;
}
11
12
13
14
15
   };
}
16
17
   int x();
   int y();
18
19 }
20
21 // 1
22 @Obj // 3*4 lines
rublic int
  return x;
}
27
28
29
30 public int y() {
```

```
31
32
                                                                       return y;
                                                                }
      39 }
40
        41 // 2
          42 @Obj
        43 interface PointSub extends Point {
4 static PointSub of(int x, int y) {
45 return new PointSub() {

           46
                                                      return x;
}
                                                         public int x() {
          47
          48
        49
50
51
52
53
54
55
                                                             public int y() {
                                                             return y;
}
                                               };
                                  }
        55 default PointSub sub(Point that) {
57 return of(this.x() - that.x(), this.y() - that.y());
58 }
59 }
        60
61 // 3
        62 @Obj
63 interface PointMul extends Point {
64 static PointMul of(int x, int y) {
65 return new PointMul() {
65 return new PointMul() {
66 return new PointMul() {
67 return new
        66
67
                                                        public int x() {
                                                        return x;
}
        68
          69
                                                             public int y() {
   return y;
}
        70
71
      71 return y;
72 }
73 };
74 }
75
76 default PointMul mul(Point that) {
77 return of(this.x() * that.x(), this.y() * that.y());
78 }

        79 }
80
          81 // 4
        82 @Obi
        82 30bp
83 interface PointDiv extends Point {
84 static PointDiv of(int x, int y) {
85 return new PointDiv() {
86 public int x() {
87 return x() {
87 return x() {
88 return x() {
87 ret
                                                      return x;
}
        87
88
        89
90
                                                         return y;
}
                                                              public int y() {
        91
92
93
94
                                };
}
        95
96
                              default PointDiv div(Point that) {
                                  return of(this.x() / that.x(), this.y() / that.y());
}
        97
98
        99 }
 100
100
101 // 5
102 @Obj // 5*6 lines
103 interface PointSumMul extends PointSum, PointMul {
104 static PointSumMul of(int x, int y) {
105 return new PointSumMul() {
106 public int x() {
107 return :
107 return 
                                                        .
return x;
}
 107
     108
 109
                                                              public int y() {
   110
 111
                                                                         return y;
                                                              }
   112
                                                 };
 113
   114 }
 115
 default PointSumMul sum(Point that) {// we have to rewrite the method to call the new "of"
return of(this.x() + that.x(), this.y() + that.y());// in order to produce an instance of PointSumMul
}
 119
 default PointSumMul mul(Point that) {
    return of(this.x() * that.x(), this.y() * that.y());
```

```
122 }
123 }
124
125 // 6
126 @Obj
127 interface PointSumSub extends PointSum, PointSub {
128 static PointSumSub of(int x, int y) {
129 return new PointSumSub() {

130
          public int x() {
          return x;
}
131
132
133
          public int y() {
134
           return y;
}
135
136
137 };
138 }
139
140 default PointSumSub sum(Point that) {// we have to rewrite the method to call the new "of"
141 return of(this.x() + that.x(), this.y() + that.y());// in order to produce an instance of PointSumMul
142 }
143
144 default PointSumSub sub(Point that) {
145 return of(this.x() - that.x(), this.y() - that.y());
146 }
147 }
148
149 // 7
150 @Obj
151 interface PointSumDiv extends PointSum, PointDiv {
152 static PointSumDiv of(int x, int y) {
153 return new PointSumDiv() {
154 public int x() {

          return x;
}
155
156
157
            public int y() {
158
           return y;
}
159
160
161
          };
162 }
102 ;
103
103
104 default PointSumDiv sum(Point that) {// we have to rewrite the method to call the new "of"
105 return of(this.x() + that.x(), this.y() + that.y());// in order to produce an instance of PointSumMul
106 }
168 default PointSumDiv div(Point that) {
169 return of(this.x() / that.x(), this.y() / that.y());
170 }
171 }
172
173 // 8
174 @Obj
175 interface PointSubMul extends PointMul, PointSub {
176
      static PointSubMul of(int x, int y) {
   return new PointSubMul() {
177
178
            public int x() {
179
              return x:
          }
180
181
182
           public int y() {
           return y;
}
183
184
185
         };
186 }
187
default PointSubMul mul(Point that) {// we have to rewrite the method to call the new "of"
return of(this.x() * that.x(), this.y() * that.y());// in order to produce an instance of PointSumMul
190 }
191
192 default PointSubMul sub(Point that) {
193 return of(this.x() - that.x(), this.y() - that.y());
194 }
195 }
196
197 // 9
198 aObi
 199 interface PointSubDiv extends PointSub, PointDiv {
200 static PointSubDiv of(int x, int y) {
201 return new PointSubDiv() {
           return x;
}
202
            public int x() {
203
204
205
            public int y() {
206
207
              return y;
            }
208
209
         };
       }
210
211
212 default PointSubDiv sub(Point that) {// we have to rewrite the method to call the new "of"
```

```
213 return of(this.x() - that.x(), this.y() - that.y());// in order to produce an instance of PointSumMul
214 }
215
216 default PointSubDiv div(Point that) {
217 return of(this.x() / that.x(), this.y() / that.y());
218 }
219 }
220
221 // 10
222 @Obj
223 interface PointMulDiv extends PointMul, PointDiv {
224 static PointMulDiv of(int x, int y) {
225 return new PointMulDiv() {
226 unblic int x() {
227 unblic int x() {
228 unblic int x() {
229 unblic int x() {
22
 226
                  public int x() {
227
                         return ×;
 228
                   }
229
                   return y;
}
 230
                     public int y() {
231
232
                }:
233
233 }
235
default PointMulDiv mul(Point that) {// we have to rewrite the method to call the new "of"
return of(this.x() * that.x(), this.y() * that.y());// in order to produce an instance of PointSumMul
}
239
249 default PointMulDiv div(Point that) {
241 return of(this.x() / that.x(), this.y() / that.y());
242 }
243 }
244
245 // 11
246 @dbj // 7*4 lines
246 @dbj // 7*4 lines
247 interface PointSumSubDiv extends PointSumSub, PointSumDiv, PointSubDiv {
248 static PointSumSubDiv of(int x, int y) {
249 return new PointSumSubDiv() {
250
                     public int x() {
                  return x;
}
 251
252
 253
254
                    return y;
}
                     public int y() {
 255
256
 257
                 };
258 }
 259
default PointSumSubDiv sum(Point that) {
    return of(this.x() + that.x(), this.y() + that.y());
262 }
 263
264 default PointSumSubDiv sub(Point that) {
265 return of(this.x() - that.x(), this.y() - that.y());
266 }
267
268 default PointSumSubDiv div(Point that) {
269 return of(this.x() / that.x(), this.y() / that.y());
270 }
271 }
272
273 // 12
274 @Obj
274 d0bj
275 interface PointSumSubMul extends PointSumSub, PointSumMul, PointSubMul {
276 static PointSumSubMul of(int x, int y) {
277 return new PointSumSubMul() {
278 public int x() {
                  return x;
}
279
 280
281
                    return y;
}
                     public int y() {
282
283
 284
285
                };
 286 }
287
 288
 289 return of(this.x() + that.x(), this.y() + that.y());
290 }
           default PointSumSubMul sum(Point that) {
289
291
 292 default PointSumSubMul sub(Point that) {
                return of(this.x() - that.x(), this.y() - that.y());
293
           }
 294
295
 296
           default PointSumSubMul mul(Point that) {
           return of(this.x() * that.x(), this.y() * that.y());
}
297
 298
299 }
 300
301 // 13
           a0bj
303 interface PointSumMulDiv extends PointSumMul. PointMulDiv. PointSumDiv {
```

```
304 static PointSumMulDiv of(int x, int y) {
305 return new PointSumMulDiv() {
            public int x() {
306
           return x;
}
307
308
309
310
            public int y() {
 311
              return y;
            }
312
313
         };
314 }
315
316 default PointSumMulDiv sum(Point that) {
317 return of(this.x() + that.x(), this.y() + that.y());
318 }
 319
default PointSumMulDiv mul(Point that) {// we have to rewrite the method to call the new "of"
return of(this.x() * that.x(), this.y() * that.y());// in order to produce an instance of PointSumMul
}
323
default PointSumMulDiv div(Point that) {
    return of(this.x() / that.x(), this.y() / that.y());
326 }
327 }
328
329 // 14
330 aObi
330 qUDJ
331 interface PointSubMulDiv extends PointSubMul, PointMulDiv, PointSubDiv {
322 static PointSubMulDiv of(int x, int y) {
333 return new PointSubMulDiv() {
334 public int x() {
           return x;
}
335
336
337
338
           return y;
}
            public int y() {
339
340
341
         };
342 }
343
 344 default PointSubMulDiv sub(Point that) {
      return of(this.x() - that.x(), this.y() - that.y());
}
345
346
347
34/ default PointSubMulDiv mul(Point that) {// we have to rewrite the method to call the new "of"
348 return of(this.x() * that.x(), this.y() * that.y());// in order to produce an instance of PointSumMul
350 }
351
default PointSubMulDiv div(Point that) {
353 return of(this.x() / that.x(), this.y() / that.y());
354 }
353
354
355 }
356
357 // 15
338 @Obj//9*1 lines
359 interface PointSumSubMulDiv extends PointSumSubMul, PointSumMulDiv, PointSumSubDiv, PointSubMulDiv {
      static PointSumSubMulDiv of(int x, int y) {
   return new PointSumSubMulDiv() {
 360
361
362
          public int x() {
363
              return x;
           }
364
365
366
            public int y() {
367
             return y;
368
369
            }
         };
370 }
371
372 default PointSumSubMulDiv sum(Point that) {
373 return of(this.x() + that.x(), this.y() + that.y());
374 }
375
376 default PointSumSubMulDiv sub(Point that) {
377 return of(this.x() - that.x(), this.y() - that.y());
378 }
379
default PointSumSubMulDiv mul(Point that) {// we have to rewrite the method to call the new "of"
return of(this.x() * that.x(), this.y() * that.y());// in order to produce an instance of PointSumMul
382 }
 383
384 default PointSumSubMulDiv div(Point that) {
       return of(this.x() / that.x(), this.y() / that.y());
}
 385
386
387 }
```

C.1.3 Scala

1 **trait** tPointState {//5

```
2 type p <: tPointState
3 def x: Int</pre>
     4 def y: Int
     5 def of(x:Int,y:Int):p
    6 }
   8 trait tPointSum extends tPointState {//3*4
9 def sum(that:p)=
10 this.of(this.x+that.x,this.y+that.y)
11 }
12
 13 trait tPointSub extends tPointState {
14 def sub(that:p)=
15 this.of(this.x-that.x,this.y-that.y)
16 }
 17
18 trait tPointMul extends tPointState {
 19 def mul(that:p)=
                   this.of(this.x*that.x,this.y*that.y)
20
21 }
22
23 trait tPointDiv extends tPointState {
24 def div(that:p)=
25 this s(this v(that v this v(that v))
25
26 }
                     this.of(this.x/that.x,this.y/that.y)
27 //glue code from now on
28 class Point(val x:Int, val y:Int) extends tPointState {//4*16
29 override type p = Point
30 override def of(x:Int,y:Int)=
31 new Point(x,y)// I have to repeat the name of the class 3 times
32 }
33
34 class PointSum(val x:Int, val y:Int) extends tPointSum {
35 override type p = PointSum
36 override def of(x:Int,y:Int)=
37
                   new PointSum(x,y)
38 }
39
40 class PointSub(val x:Int, val y:Int) extends tPointSub {
41 override type p = PointSub
42 override def of(x:Int,y:Int)=
43 new PointSub(x,y)
44 }
45
46 class PointMul(val x:Int, val y:Int) extends tPointMul {
47 override type p = PointMul
48 override def of(x:Int,y:Int)=
49 new PointMul(x,y)
 50 }
51
52 class PointDiv(val x:Int, val y:Int) extends tPointDiv {
53 override type p = PointDiv
54 override def of(x:Int,y:Int)=

55
                   new PointDiv(x,y)
56 }
57
58 class PointSumSub(val x:Int, val y:Int) extends tPointSum with tPointSub {
59 override type p = PointSumSub
60 override def of(x:Int,y:Int)=
61 override def of(x:Int,y:Int)=
63 override type p = PointSumSub (p = PointSum (p
61
            new PointSumSub(x,y)
62 }
63
63
64 class PointSumMul(val x:Int, val y:Int) extends tPointSum with tPointMul {
65 override type p = PointSumMul
66 override def of(x:Int,y:Int)=
67 new PointSumMul(x,y)

67
68 }
69
70 class PointSumDiv(val x:Int, val y:Int) extends tPointSum with tPointDiv {
71 override type p = PointSumDiv
72 override def of(x:Int,y:Int)=
73 new PointSumDiv(x,y)
74 }
 75
76 class PointSubMul(val x:Int, val y:Int) extends tPointSub with tPointMul {
77 override type p = PointSubMul
78 override def of(x:Int,y:Int)=
79 override def of(x:Int,y:Int)=
70 override def of(x:Int,y:Int)=
71 override def of(x:Int,y:Int)=
72 override def of(x:Int,y:Int)=
73 override def of(x:Int,y:Int)=
74 override def of(x:Int,y:Int)=
75 override def 
 79
                   new PointSubMul(x,y)
80 }
81
82 class PointSubDiv(val x:Int, val y:Int) extends tPointSub with tPointDiv {
83 override type p = PointSubDiv
84 override def of(x:Int,y:Int)=
85 new PointSubDiv(x,y)
86 }
87
88 class PointMulDiv(val x:Int, val y:Int) extends tPointMul with tPointDiv {
89 override type p = PointMulDiv
90 override def of(x:Int,y:Int)=
91
92 }
                    new PointMulDiv(x,y)
```

```
93
 94 class PointSumSubDiv(val x:Int, val y:Int) extends tPointSum with tPointSub with tPointDiv {
 95 override type p = PointSumSubDiv
96 override def of(x:Int,y:Int)=
 97
      new PointSumSubDiv(x,y)
 98 }
 99
100 class PointSumSubMul(val x:Int, val y:Int) extends tPointSum with tPointSub with tPointMul {
101 override type p = PointSumSubMul
102 override def of(x:Int,y:Int)=
103 new PointSumSubMul(x,y)
104 }
105
106 class PointSumMulDiv(val x:Int, val y:Int) extends tPointSum with tPointMul with tPointDiv {
107 override type p = PointSumMulDiv
108 override def of(x:Int,y:Int)=
      new PointSumMulDiv(x,y)
109
110 }
111
112 class PointSubMulDiv(val x:Int, val y:Int) extends tPointSub with tPointMul with tPointDiv {
113 override type p = PointSubMulDiv
114 override def of(x:Int,y:Int)=
115 new PointSubMulDiv(x,y)
116 }
117
118 class PointSumSubMulDiv(val x:Int, val y:Int) extends tPointSum with tPointSub with tPointMul with tPointDiv {
119 override type p = PointSumSubMulDiv
120 override def of(x:Int,y:Int)=
121
      new PointSumSubMulDiv(x,y)
121
```

C.1.4 42

The main differences with respect to 42_{μ} as presented in the paper are: we use Resource to declare a trait, all trait names start with \$, traits are referred to with a 'O'. Method bodies can be simple expressions, without the need for a 'return'. Note that in full 42 there is no primitive type int, here we use Num instead, which is an arbitrary precision rational.

```
1 $p: Resource <>< {//4
  2 method Num x()
3 method Num y()
     class method This of(Num x,Num y)
  4
  6
  7 $pointSum: Resource <>< Use[$p()] <>< {//3*4
     method This sum(This that)
This.of(x: this.x()+that.x(), y: this.y()+that.y())
 8
10 }
 11
12 $pointSub: Resource <>< Use[$p()] <>< {</pre>
 13
    method This sub(This that)
14
      This.of(x: this.x()-that.x(), y: this.y()-that.y())
 15
    }
16
17 $pointMul: Resource <>< Use[$p()] <>< {
18 method This mul(This that)</pre>
 19
       This.of(x: this.x()*that.x(), y: this.y()*that.y())
20 }
sepulation: Resource <>< Use[$p()] <>< {
    method This div(This that)
    This.of(x: this.x()/that.x(), y: this.y()/that.y())
    S
}</pre>
27 Point: Use[$p()] <>< {}//1*16
28
29 PointSum: Use[$pointSum()] <>< {}</pre>
 30
31 PointSub: Use[$pointSub()] <>< {}
32</pre>
33 PointMul: Use[$pointMul()] <>< {}</pre>
 34
35 PointDiv: Use[$pointDiv()] <>< {}</pre>
 36
37 PointSumSub: Use[$pointSum();$pointSub()] <>< {}
38</pre>
39 PointSumMul: Use[$pointSum();$pointMul()] <>< {}</pre>
 40
41 PointSumDiv: Use[$pointSum();$pointDiv()] <>< {}</pre>
 42
 43 PointSubMul: Use[$pointSub();$pointMul()] <>< {}</pre>
45 PointSubDiv: Use[$pointSub();$pointDiv()] <>< {}</pre>
```

46	
47	PointMulDiv: Use [\$pointMul();\$pointDiv()] <>< {}
48	
49	PointSumSubDiv: Use [\$pointSum();\$pointSub();\$pointDiv()] <>< {}
50	
51	PointSumSubMul: Use [\$pointSum();\$pointSub();\$pointMul()] <>< {}
52	
53	PointSumMulDiv: Use [\$pointSum();\$pointMul();\$pointDiv()] <>< {}
54	
55	PointSubMulDiv: Use [\$pointSub();\$pointMul();\$pointDiv()] <>< {}
56	
57	<pre>PointSumSubMulDiv: Use[\$pointSum();\$pointSub();\$pointMul();\$pointDiv()] <>< {}</pre>

C.2 FCPoint

C.2.1 Java7

```
2 NONE, SOUR, SWEET, SALTY, SPLICY;
3 }
        5 class Color {
        6 final int r;
7 final int g;
         8 final int b;
         9
    public Color(int r, int g, int b) {
    this.r = r;
                    this.r = r;
this.g = g;
   this.1 = 1,

this.g = g;

this.b = b;

this.b = b;

this.b = b;

this.b = b;
    16 public Color mix(Color that) {
17 return new Color((this.r + that.r) / 2, (this.g + that.g) / 2, (this.b + that.b) / 2);
18 }
    18
19 }
    20
21 class Point {//10 lines
    22 final int x;
23 final int y;
    24
25 public Point(int x, int y) {
public Point(
    this.x = x;
    this.y = y;
    }
    29
    30

  29
30 public Point withX(int that) {
31 return new Point(that, this.y);
32 }
33
34 public Point withY(int that) {
35 return new Point(this.x, that);
36 }
37 }
38 class PointSum extends Point {//9 ]
39 class PointSum extends Point {//9 ]
30 class PointSum extends Point {
30 class PointSum extends PointSum ext
    39 class PointSum extends Point {//9 lines
40 public PointSum(int x, int y) {
    41 super(x, y);
42 }
 42 }
43
44 public PointSum withX(int that) {
45 return new PointSum(that, this.y);
46 }
47
48 public PointSum withY(int that) {
49 return new PointSum(this.x, that);

    49 return new PointSum(this.x, that);
50 }
   51
52 public PointSum sum(Point that) {
52 public withX(this.x + that.
                   return this.withX(this.x + that.x).withY(this.y + that.y);
}
    53
54
   55 }
56
57 class CPoint extends PointSum {//13 lines
58 final Color color;
59
     60
                 public CPoint(int x, int y, Color color) {
    61
62
                        super(x, y);
this.color = color;
                 }
    63
64
    bis public CPoint withX(int that) {
    for return new CPoint(that, this.y, this.color);
    }

     68
    69 public CPoint withY(int that) {
```

```
70 return new CPoint(this.x, that, this.color);
71 }
 72
 73 public CPoint withColor(Color that) {
 74 return new CPoint(this.x, this.y, that);
75 }
 76
    return this.withColor(this.color.mix(that.color));
}
 77 public CPoint merge(CPoint that) {
 78
 79
 80 }
 81
 82 class FCPoint extends CPoint {//15 lines
83 final Flavor flavor;
 84
 85
    public FCPoint(int x, int y, Color color, Flavor flavor) {
      super(x, y, color);
this.flavor = flavor;
 86
 87
     }
 88
 89
    public FCPoint withX(int that) {
    return new FCPoint(that, this.y, this.color, this.flavor);
}
 90
 91
92
 93
94
    public FCPoint withY(int that) {
 95 return new FCPoint(this.x, that, this.color, this.flavor);
96 }
 97
98 public FCPoint withColor(Color that) {
99 return new FCPoint(this.x, this.y, that, this.flavor);
100 }
101
102 public FCPoint withFlavor(Flavor that) {
     return new FCPoint(this.x, this.y, this.color, that);
}
103
104
105
106 public FCPoint merge(FCPoint that) {
       return this.withColor(that.color).withFlavor(that.flavor);
107
     }
108
109 }
```

C.2.2 Scala

```
1 trait tPointState {//8 lines
  2
     type p <: tPointState
def x: Int</pre>
 3
     def y: Int
  4
 5 def withX(that:Int):p
6 def withY(that:Int):p
 7
     def of(x:Int,y:Int):p
 8 def merge(that:p):p
 9 }
10
11 trait tPointSum extends tPointState {//3
12 def sum(that:p)=
13 this.merge(that).withX(this.x+that.x)
14 }
      this.merge(that).withX(this.x+that.x).withY(this.y+that.y)
15
16 trait tColored {//6*2
17 type p <: tColored
18 def color:Int
19 def withColor(that:Int):p
20 def merge(that:p)=</pre>
21 this.withColor(this.color+that.color)
22 }
23
24 trait tFlavored{
25 type p <: tFlavored
26 def flavor:Int
27 def withFlavor(that:Int):p
28 def merge(that:p)=
      this.withFlavor(that.flavor)
29
 30
42 def withColor(that:Int)=
    of(x,y,that)
43
44
```

```
46 class FCPoint(val x:Int, val y:Int, val color:Int, val flavor:Int) extends tPointSum with tColored with tFlavored{
47 override type p = FCPoint
48 def of(x:Int,y:Int)=
49 this.of(x,y,0,0)
50 def of(x:Int,y:Int,color:Int,flavor:Int)=
51 new FCPoint(x,y,color,flavor)
52 def withX(that:Int)=
53 of(that,y,color,flavor)
54 def withY(that:Int)=
55 of(x,that,color,flavor)
56 def withColor(that:Int)=
57 of(x,y,that,flavor)
58 def withFlavor(that:Int)=
59 of(x,y,color,that)
60 def superMergeFlavoured(that:FCPoint)=
51 super[tFlavored].merge(that).superMergeFlavoured(that)
```

C.2.3 42

64 }

```
1 Flavor: Enumeration"sour, sweet, salty, spicy"
 3 Color:Data<><{Num r Num g Num b
    method This mix(This that)
       Color( r:(this.r()+that.r())/2Num, g:(this.g()+that.g())/2Num, b:(this.b()+that.b())/2Num)
 5
 6
     }
 8 $p: Resource <>< {// 7 lines
9 method Num x()
10 method Num y() //getters
11 method This withX(Num that)
method This withY(Num that)//withers
class method This of(Num x,Num y)
14 method This merge(This that) //new method merge!
15 }
13 f
16
17 $pointSum: Resource <>< Use[$p()] <>< {// 3 lines
18 method This sum(This that)
19 this.merge(that).withX(this.x()+that.x()).withY(this.y()+that.y())
20 }</pre>
21
22 $colored: Resource <>< {// 5 *2 lines
23 method Color color()
24 method This withColor(Color that)
25 method This merge(This that) //how to merge colors
26
       this.withColor(this.color().mix(that.color()))
    }
27
28
35
36 CPoint: Use[$pointSum();$colored()] <>< {// 4 lines
37 class method This of(Num x,Num y)
38 This.of(x:x,y:y,color:Color(r:100Num,g:0Num,b:0Num))
39 class method This of(Num x, Num y,Color color)
40
41
42 FCPoint: Use[//9 lines
     Refactor2.toAbstract(selector:\"merge(that)", into:\"_1merge(that)")<><$colored();
Refactor2.toAbstract(selector:\"merge(that)", into:\"_2merge(that)")<><$flavored();
43
44
45 $pointSum()] <>< {
46 class method This of(Num x,Num y)</pre>
       This.of(x:x,y:y,color:Color(r:100Num,g:0Num,b:0Num),flavor:Flavor.sour())
class method This of(Num x, Num y,Color color,Flavor flavor)
47
48
49
50
        method This merge(This that)
          this._1merge(that)._2merge(that)
51
       3
```

C.3 Expression problem

C.3.1 Original Scala

Here is the original code of Scala solving the expression problem. As you can see, sometimes multiple operations/cases are declared together in the same trait.

1 **trait** Base {

```
2 type exp <: Exp
 4 trait Exp {
 5 def eval: Int
6 }
8 class Num(val value: Int) extends Exp {
9 def eval: Int = value
10 }
12 type BaseNum = Num
13 }
11
14
14
15 trait BasePlus extends Base {
16 class Plus(val left: exp, val right: exp) extends Exp {
17 def eval: Int = left.eval + right.eval
18 }
19
20 type BasePlus = Plus
21 }
22
22 trait BaseNeg extends Base {
23 trait BaseNeg extends Base {
24 class Neg(val term: exp) extends Exp {
25 def eval = -term.eval
26 }
27
28 type BaseNeg = Neg
29 }
30 trait BasePlusNeg extends BasePlus with BaseNeg
31 trait Show extends Base {
32 type exp <: Exp
33
34 trait Exp extends super.Exp {
35 def show: String
36 }
37
38 trait NumBehavior extends Exp {
self: BaseNum =>
override def show: String = value.toString
}
42
43
44 }
    final class Num(v: Int) extends BaseNum(v) with NumBehavior with Exp
45
46 trait ShowPlusNeg extends BasePlusNeg with Show {
47 trait PlusBehavior {
48
     self: BasePlus =>
       def show = left.show + "+" + right.show;
49
50
    }
51
52 final class Plus(l: exp, r: exp) extends BasePlus(l, r) with PlusBehavior with Exp
53
54 trait NegBehavior {
55
     self: BaseNeg =>
def show = "-(" + term.show + ")";
56
57 }
58
59
    class Neg(t: exp) extends BaseNeg(t) with NegBehavior with Exp
60 }
61
62 trait DblePlusNeg extends BasePlusNeg {
63 type exp <: Exp
64
65 trait Exp extends super.Exp {
   def dble: exp
}
66
67
68
69 def Num(v: Int): exp
70
71 def Plus(l: exp, r: exp): exp
72
73 def Neg(t: exp): exp
74
75
    trait NumBehavior {
76
77
       self: BaseNum =>
def dble = Num(value * 2)
    }
78
79
80 final class Num(v: Int) extends BaseNum(v) with NumBehavior with Exp
81
82 trait PlusBehavior {
      self: BasePlus =>
def dble = Plus(left.dble, right.dble)
83
84
85
    }
86
87 class Plus(l: exp, r: exp) extends BasePlus(l, r) with PlusBehavior with Exp
88
89 trait NegBehavior {
90
       self: BaseNeg =>
91
92 }
       def dble = Neg(term.dble)
```

```
93
    class Neg(t: exp) extends super.Neg(t) with NegBehavior with Exp
 94
 95 }
 96 //-- 52 lines up to here, not counting new lines and '}'
97 trait Equals extends Base {
98 type exp <: Exp;
 99
100 trait Exp extends super.Exp {
101
     def eql(other: exp): Boolean;
102
103
104 }
      def isNum(v: Int): Boolean = false;
105
106 trait NumBehavior extends Exp {
107
       self: BaseNum =:
      def eql(other: exp): Boolean = other.isNum(value);
108
109
      override def isNum(v: Int) = v == value;
110
111 }
112
113 final class Num(y: Int) extends BaseNum(y) with NumBehavior with Exp
114 }
115
116 trait EqualsPlusNeg extends BasePlusNeg with Equals {
117 type exp <: Exp;</pre>
118
trait Exp extends super[BasePlusNeg].Exp
with super[Equals].Exp {
      def isPlus(l: exp, r: exp): Boolean = false;
121
122
      def isNeg(t: exp): Boolean = false;
123
124 }
125
final class Num(v: Int) extends BaseNum(v)
with NumBehavior // effectively super[Equals].NumBehavior
128
      with Exp
129
130 trait PlusBehavior extends Exp {
131
      self: BasePlus =>
def eql(other: exp): Boolean = other.isPlus(left, right);
132
133
134
      override def isPlus(l: exp, r: exp) = (left eql l) && (right eql r)
135
    3
136
137
     final class Plus(l: exp, r: exp) extends BasePlus(l, r) with PlusBehavior with Exp
138
139 trait NegBehavior extends Exp {
      self: BaseNeg =>
def eql(other: exp): Boolean = other.isNeg(term);
140
141
142
143
      override def isNeg(t: exp) = term eql t
144 }
145
     final class Neg(t: exp) extends BaseNeg(t) with NegBehavior with Exp
146
147 }
148
149 trait EqualsShowPlusNeg extends EqualsPlusNeg with ShowPlusNeg {
150 type exp <: Exp
151
152 trait Exp extends super[EqualsPlusNeg].Exp
153
       with super[ShowPlusNeg].Exp
154
156 self: BaseNum =>
157 }
157
155 trait NumBehavior extends super[EqualsPlusNeg].NumBehavior with super[ShowPlusNeg].NumBehavior {
158
159 class Num(v: Int) extends BaseNum(v) with NumBehavior with Exp
160
161 trait PlusBehavior extends super[EqualsPlusNeg].PlusBehavior with super[ShowPlusNeg].PlusBehavior {
162 self: BasePlus =>
163 }
164
165 class Plus(l: exp, r: exp) extends BasePlus(l, r) with PlusBehavior with Exp
166
167 trait NegBehavior extends super[EqualsPlusNeg].NegBehavior with super[ShowPlusNeg].NegBehavior {
168 self: BaseNeg =>
    self: BaseNeg =>
}
169
170
171 class Neg(term: exp) extends BaseNeg(term) with NegBehavior with Exp
172
173 //--- 40 lines for equals
```

C.3.2 Isolated Scala

The following code is fully modularized: every trait defines exactly one operation for each datavariant.

```
1 trait Base {
2 type exp <: Exp
   3
  4 trait Exp { }
5 }
  6
  7 trait BaseNum extends Base {
   8 class Num(val value: Int) extends Exp { }
  9
 10 type BaseNum = Num
11 }
 12
13 trait BasePlus extends Base {
14 class Plus(val left: exp, val right: exp) extends Exp { }
15
16 type BasePlus = Plus
17 }
 18
19 trait BaseNeg extends Base {
20 class Neg(val term: exp) extends Exp { }
 21
22 type BaseNeg = Neg
23 }
24 //----EVAL
25 trait Eval extends Base {
 26 type exp <: Exp
27
28 trait Exp extends super.Exp {
29 def eval: Int
30 }
 31 }
32
33 //--
                                                              --EVALNUM
33 //----EVALNUM
33 trait EvalNum extends BaseNum with Eval {
35 trait NumBehavior {
36 self: BaseNum =>
37 def eval: Int = value
38 }
 39
 40 class Num(v: Int) extends BaseNum(v) with NumBehavior with Exp
 41 }
 42
 43 //----EVALPLUS
44 trait EvalPlus extends BasePlus with Eval {
 45 trait PlusBehavior {
46 self: BasePlus =>
47 def eval = left.eval + right.eval;
48 }
 49
       class Plus(l: exp, r: exp) extends BasePlus(l, r) with PlusBehavior with Exp
 50
51 }
52 //----EVALNEG
 53 trait EvalNeg extends BaseNeg with Eval {
54 trait NegBehavior {
55 self: BaseNeg =>
56
57 }
58
             def eval = - term.eval;
59 class Neg(t: exp) extends BaseNeg(t) with NegBehavior with Exp
60 }
61 //-----SHOW
62 trait Show extends Base {
63 type exp <: Exp
64
65 trait Exp extends super.Exp {
66 def show: String
67 }
68 def show: String
67 }
68 def show: String
67 }
68 def show: String
69 def show: String
69 def show: String
69 def show: String
69 def show: String
60 def show: 
67
68 }
 69 //----SHOWNUM
 70 trait ShowNum extends BaseNum with Show {
71 trait NumBehavior {
72 self: BaseNum =>
73 def show: String = value.toString
74 }
 75
 76 class Num(v: Int) extends BaseNum(v) with NumBehavior with Exp
 77 }
 78
 79 //-----SHOWPLUS
 80 trait ShowPlus extends BasePlus with Show {
80 trait Showrus extends basertus with Show {
81 trait PlusBehavior {
82 self: BasePlus =>
83 def show = left.show + "+" + right.show;
84 }
95
 85
 86
        class Plus(l: exp, r: exp) extends BasePlus(l, r) with PlusBehavior with Exp
87 }
 88 //-----SHOWNEG
 89 trait ShowNeg extends BaseNeg with Show {
90 trait NegBehavior {
91 self: BaseNeg =>
```

```
92 def show = "-(" + term.show + ")";
93 }
 94
 95
     class Neg(t: exp) extends BaseNeg(t) with NegBehavior with Exp
 96 }
 97 //----
                 -----DBLE
 98 trait Dble extends Base {
 99 type exp <: Exp
100
    def dble: exp
}
101 trait Exp extends super.Exp {
102
103
104 }
105 //----
           -----DBLENUM
106 trait DbleNum extends BaseNum with Dble {
107 type exp <: Exp
108
109 trait Exp extends super[BaseNum].Exp with super[Dble].Exp
110
111 trait NumBehavior {
112
        self: BaseNum =>
112 Set1: BaseNum =>
113 def dble = Num(value * 2)
114 }
115
116 def Num(v: Int): exp
117
118 class Num(v: Int) extends super.Num(v) with NumBehavior
119 }
120 //--
                   -----DBLEPLUS
121 trait DblePlus extends BasePlus with Dble {
122 type exp <: Exp</pre>
123
124 trait Exp extends super[BasePlus].Exp with super[Dble].Exp
125
127 self: BasePlus =>
128 def dble = Plus(left.dble, right.dble)
129 }
126 trait PlusBehavior {
130
131 def Plus(l: exp, r: exp): exp
132
133 class Plus(l: exp, r: exp) extends super.Plus(l, r) with PlusBehavior with Exp
134 }
135 //--
                             --- DBL ENEG
136 trait DbleNeg extends BaseNeg with Dble {
137 type exp <: Exp
138
139 trait Exp extends super[BaseNeg].Exp with super[Dble].Exp
140
141 trait NegBehavior {
      self: BaseNeg =>
142
143
144 }
       def dble = Neg(term.dble)
145
146 def Neg(t: exp): exp
147
148 class Neg(t: exp) extends super.Neg(t) with NegBehavior with Exp
149 }//78 lines up to here, not couting new lines and '}'
150
151

151
152 //glue code: 27 lines
153 object All0 extends
154 Eval with Show with Dble with
155 EvalNum with EvalPlus with EvalNeg with

156
       ShowNum with ShowPlus with ShowNeg with DbleNum with DblePlus with DbleNeg {
157
158 override type exp = Exp
159
160 trait Exp extends
         super[Eval].Exp with super[Show].Exp with super[Dble].Exp with
161
       Super[DbleNum].Exp with super[DblePlus].Exp with super[DbleNeg].Exp
162
163
164 trait NumBehavior extends
165 super[EvalNum].NumBehavior with super[ShowNum].NumBehavior with
       super[DbleNum].NumBehavior {
  self: BaseNum =>
166
167
168 }
169
170 class Num(v: Int) extends BaseNum(v) with NumBehavior with Exp
171
172
     trait PlusBehavior extends
         super[EvalPlus].PlusBehavior with super[ShowPlus].PlusBehavior with
super[DblePlus].PlusBehavior {
173
174
175
        self: BasePlus =>
     }
176
177
      class Plus(l: exp, r: exp) extends BasePlus(l, r) with PlusBehavior with Exp
178
179
180 trait NegBehavior extends
         super[EvalNeg].NegBehavior with super[ShowNeg].NegBehavior with
super[DbleNeg].NegBehavior {
181
182
```

```
183 self: BaseNeg =>
184 }
185
186 class Neg(t: exp) extends BaseNeg(t) with NegBehavior with Exp
187
188 def Num(v: Int) = new Num(v)
189
190 def Plus(l: exp, r: exp) = new Plus(l,r)
191
192
     def Neg(t: exp) = new Neg(t)
193 }
194 //----
            -----EQUALS
195 trait Equals extends Base {
196 type exp <: Exp;</pre>
197
198 trait Exp extends super.Exp {
199 def eql(other: exp): Boolean;
199
200 }
201 }
202 //----
              -----EQUALSNUM
203 trait EqualsNum extends BaseNum with Equals {
204 type exp <: Exp;
205
206 trait Exp extends super.Exp {
207 def isNum(v: Int): Boolean = false;
208 }
209
210 trait NumBehavior extends Exp {
211
        self: BaseNum =>
      def eql(other: exp): Boolean = other.isNum(value);
212
213
214 override def isNum(v: Int) = v == value;
215 }
216
217
     class Num(v: Int) extends BaseNum(v) with NumBehavior with Exp
218 }
219 //---
                    -----EQUALSPLUS
220 trait EqualsPlus extends BasePlus with Equals {
221 type exp <: Exp;
222
224 def isPlus(l: exp, r: exp): Boolean = false;
225 }
223 trait Exp extends super[BasePlus].Exp with super[Equals].Exp {
226
227 trait PlusBehavior extends Exp {
     self: BasePlus =>
def eql(other: exp): Boolean = other.isPlus(left, right);
228
229
231 override def isPlus(l: exp, r: exp) = (left eql l) & (right eql r)
232 }
233
234
     class Plus(l: exp, r: exp) extends BasePlus(l, r) with PlusBehavior with Exp
235 }
236 //--
                    -----
                               --EOUALSNEG
237 trait EqualsNeg extends BaseNeg with Equals {
238 type exp <: Exp;
239
240 trait Exp extends super[BaseNeg].Exp with super[Equals].Exp {
241 def isNeg(t: exp): Boolean = false;
242 }
243
244 trait NegBehavior extends Exp {
245 self: BaseNeg =>
246 def eql(other: exp): Boolean = other.isNeg(term);
247
       override def isNeg(t: exp) = term eql t
248
249 }
250
251 class Neg(t: exp) extends BaseNeg(t) with NegBehavior with Exp
252 }
252 J
253 //31 lines for equals, 29 glue

254 object All extends
255 Eval with Show with Dble with Equals with
256 EvalNum with EvalPlus with EvalNeg with

       ShowNum with ShowPlus with ShowNeg with
DbleNum with DblePlus with DbleNeg with
257
258
259
       EqualsNum with EqualsPlus with EqualsNeg
260 {
261
      override type exp = Exp
262
263 trait Exp extends
        super[Eval].Exp with super[Show].Exp with super[Dble].Exp with super[Equals].Exp with
super[DbleNum].Exp with super[DblePlus].Exp with super[DbleNeg].Exp with
super[EqualsNum].Exp with super[EqualsPlus].Exp with super[EqualsNeg].Exp
264
265
266
267
268 trait NumBehavior extends
         super[EvalNum].NumBehavior with super[ShowNum].NumBehavior with
super[DbleNum].NumBehavior with super[EqualsNum].NumBehavior {
269
270
271
        self: BaseNum =>
272
273
     }
```

```
274 class Num(v: Int) extends BaseNum(v) with NumBehavior with Exp
275
276 trait PlusBehavior extends
277 super[EvalPlus].PlusBehavior with super[ShowPlus].PlusBehavior with
278 super[DblePlus].PlusBehavior with super[EqualsPlus].PlusBehavior {
279
       self: BasePlus =>
280 }
281
282 class Plus(l: exp, r: exp) extends BasePlus(l, r) with PlusBehavior with Exp
283
284
     trait NegBehavior extends
285
        super[EvalNeg].NegBehavior with super[ShowNeg].NegBehavior with
       super[DbleNeg].NegBehavior with super[EqualsNeg].NegBehavior {
  self: BaseNeg =>
286
287
288 }
289
290 class Neg(t: exp) extends BaseNeg(t) with NegBehavior with Exp
291
292 def Num(v: Int) = new Num(v)
293
294 def Plus(l: exp, r: exp) = new Plus(l,r)
295
     def Neg(t: exp) = new Neg(t)
296
297 }
```

C.3.3 42

The classes RenNum, RenPlus, and RenNeg can be understood as declarations of short cuts in order to make the full 42 language more in line with the more compact syntax of 42_{μ} . In 42, these short cuts can be very expressive but here we use them only to better align the full 42 and 42_{μ} languages.

```
1 RenNum:{class method Library<>< (Library that) exception Guard
2 Refactor2.rename(path:\"T" into:\"Num")<><that</pre>
 3
 8
9 RenNeg:{class method Library<>< (Library that) exception Guard
10 Refactor2.rename(path:\"T" into:\"Neg")<><that</pre>
11 }
12 //above, header not counted
13
14 $exp:Resource<><{ //3 lines
15
    Exp:{interface}
16
17 T:{implements Exp}
18 }
19
20 $num:Resource<><Use[RenNum<><$exp()]<><{ //4 lines
21 Num:{
21 Nulm:1
22 method Size value()
23 class method Num of(Size value)
24
25 }
26
27 $plus:Resource<><Use[RenPlus<><$exp()]<><{ //6 lines
28
29
     Exp:{interface}
30 Plus:{
31 metho
      method Exp left()
      method Exp tert()
method Exp right()
class method Plus of(Exp left, Exp right)
32
33
34
35 }
36
37 $neg:Resource<><Use[RenNeg<><$exp()]<><{ //5 lines
38
39
40 Neg:{
41 method Exp term()
42
43
       class method Neg of(Exp term)
     }
44 }
45
46 $eval:Resource<>< { //4 lines
47 Exp:{interface</pre>
48
49
      method Size eval()
}
50
51 T:{implements Exp}
52
53
```

```
54 $evalNum:Resource<><Use[$num(); RenNum<><$eval()]<>< { // 2*3 lines
55 Num:{ method Size eval() this.value() }</pre>
  56
           }
  57
 58 $evalPlus:Resource<><Use[$plus(); RenPlus<><$eval()]<>< {
59 Plus:{ method Size eval() this.left().eval()+this.right().eval() }
</pre>
  60 }
  61
 62 $evalNeg:Resource<><Use[$neg(); RenNeg<><$eval()]<>< {
63 Neg:{ method Size eval() Size"-1" * this.term().eval() }</pre>
  64 }
  65
  66 $show:Resource<><Use[$exp()]<><{ //4 lines
  67
        Exp:{interface
            method S show()
  68
  69
  70
  71 T:{implements Exp}
  72
          }
  73
 74 $showNum:Resource<><Use[$num(); RenNum<><$show()]<><{ //2*3 lines
75 Num:{ method S show() this.value().toS() }</pre>
 75
76
         }
 77
78 $showPlus:Resource<><Use[$plus(); RenPlus<><$show()]<><{</pre>
  79 Plus:{ method S show() this.left().show()++S" + "++this.right().show() }
  80 }
  81
 82 $showNeg:Resource<><Use[$neg(); RenNeg<><$show()]<><{
83 Neg:{ method S show() S"-("++this.term().show()++S")" }</pre>
  84
  85 //--
  86 $double:Resource<><{ //4 lines
           method Exp double()
}
  87
        Exp:{interface
  88
  89
  90
  91 T:{implements Exp}
  92
  93
  94 $doubleNum:Resource<><Use[$num(); RenNum<><$double()]<><{ //2*3 lines
  95
        Exp:{interface}//not counted, needed in the full language only to guide scope resolution in the desugaring
  96
  97 Num:{ method Exp double() Num.of(value: this.value()*2Size) }
  98 }
 99
100 $doublePlus:Resource<><Use[$plus(); RenPlus<><$double()]<><{
101 Exp:{interface}
102
103 Plus:{ method Exp double() Plus.of(left: this.left().double(), right: this.right().double()) }
104
105
106 $doubleNeg:Resource<><Use[$neg(); RenNeg<><$double()]<><{
107 Exp:{interface}
108
109 Neg:{ method Exp double() Neg.of(term: this.term().double()) }
110 }
111 //----
112 $equals:Resource<><Use[$eval()]<><{ //6 lines</pre>
113 Exp:{interface
internet interne
116
117
118 T:{implements Exp
119 method eql(that) that.equalToT(this)
120 }
121 }
122
123 $equalsNum:Resource<><Use[$num(); //5*3 lines</pre>
124 Refactor2.Method[rename:\"equalToT(that)" of:\"Exp" into:\"equalToNum(that)"]
125 <><RenNum<>< $equals()]<><{</pre>
126 Num:{
            method Bool equalToNum(Num that) this.value()==that.value()
127
128
             }
129 }
130
131 $equalsPlus:Resource<><Use[$plus();</pre>
132 Refactor2.Method[rename:\"equalToT(that)" of:\"Exp" into:\"equalToPlus(that)"]
133 <><RenPlus<><$equals()]<><{</pre>
134 Plus:{
135 method Bool equalToPlus(Plus that)
136 this.left().eql(that.left()) & this.right().eql(that.right())
137
            }
138 }
139
140 $equalsNeg:Resource<><Use[$neg():
141 Refactor2.Wethod[rename:\"equalToT(that)" of:\"Exp" into:\"equalToNeg(that)"]
142 <><RenNeg<><$equals()]<><{</pre>
143 Neg:{
             method Bool equalToNeg(Neg that) this.term().egl(that.term())
144
```

```
145 }
146 }
147
148
      //16+6 lines glue code
149 $equalsAll:Resource<><Use[$equalsNeg();$equalsPlus();$equalsNum()]<><{
149 $equalsAll:Resource<><Use[$equalsNeg()
150 Exp:{interface
151 method Bool eql(Exp that)
152 method Bool equalToNum(Num that)
153 method Bool equalToPlus(Plus that)
154 method Bool equalToNeg(Neg that)
155</pre>
155
       method Size eval()

156
157
157
158 Num:{implements Exp
159 method equalToPlus(that) Bool.false()
       method equalToNeg(that) Bool.false()
160
161
162
162
163 Plus:{implements Exp
164 method equalToNum(that) Bool.false()
165
       method equalToNeg(that) Bool.false()
166
167
168 Neg:{implements Exp
169 method equalToNum(that) Bool.false()
169
internod equalToPlus(that) Bool.false()
method equalToPlus(that) Bool.false()
171
172 }
         }
173
174 $evalAll:Resource<><Use[$evalPlus();$evalNeg()]<><$evalNum()
175
176 $showAll:Resource<><Use[$showNum();$showPlus()]<><$showNeg()
177
178 $doubleAll:Resource<><Use[$doubleNum();$doublePlus()]<><$doubleNeg()
179
180 ESDAll0:Use[Use[$evalPlus();$evalNeg()]<><$evalNum();
181 Use[$showNum();$showPlus()]<><$showNeg();
182 $doubleNum();$doublePlus()]<><$doubleNeg()</pre>
183
184
185 ESDAll:Use[$evalAll();$showAll()]<><$doubleAll()</pre>
186
187 ESDEAll:Use[$evalAll();$showAll();$doubleAll()]<><$equalsAll()</pre>
188
189 $equals2:Resource<><{ //7 lines
109 Exp:{interface method Bool equals(Exp that)}
191 T:{implements Exp
192 method Bool exactEquals(T that)
192 method Bool exactEquals(T that)
193 method equals(that){
194 with that (on T return this.exactEquals(that) )
195 return Bool.false()
197
196
          }
197
198 }
        }
199
200 $equalsNum2:Resource<><Use[$num();RenNum<><$equals2()]<><{ //2*3 lines
201 Num:{ method Bool exactEquals(Num that) this.value().equals(that.value()) }
202 }
203
204 $equalsPlus2:Resource<><Use[$plus();RenPlus<><$equals2()]<><{
205 Plus:{ method Bool exactEquals(Plus that) this.left().equals(that.left()) & this.right().equals(that.right()) }
206 }
207
208 $equalsNeg2:Resource<><Use[$neg();RenNeg<><$equals2()]<><{
209 Neg:{method Bool exactEquals(Neg that) this.term().equals(that.term()) }
210 }
211
212 $equalsAll2:Resource<><Use[$equalsNum2();$equalsPlus2()]<><$equalsNeg2()</pre>
213
214 ESDEAll2:Use[$evalAll();$showAll();$doubleAll()]<><$equalsAll2()</pre>
```